AD 746513

(11)

PAPER P-728

THE TRACK-ESTABLISHMENT PROCESS IN A MIXED FALSE-ALARM-RATE ENVIRONMENT

Robert D. Turner
Stanley Marder

June 1972

IDA
INSTITUTE FOR DEFENSE ANALYSES
SCIENCE AND TECHNOLOGY DIVISION

IDA Log No. HQ 72-13898
Copy 50 of 60 copies

PAGES ______ ARE MISSING IN ORIGINAL DOCUMENT

The work reported in this document was conducted under contract DAHC15 67 C 0011 for the Department of Defense. The publication of this IDA Paper does not indicate endorsement by the Department of Defense, nor should the contents be construed as reflecting the official position of that agency.

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) INSTITUTE FOR DEFENSE ANALYSES 400 Army-Navy Drive Arlington, Virginia 22202	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP --

3. REPORT TITLE

The Track-Establishment Process in a Mixed False-Alarm-Rate Environment

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Paper P-728, June 1972

5. AUTHOR(S) (First name, middle initial, last name)

Robert D. Turner, Stanley Marder

6. REPORT DATE June 1972	7a. TOTAL NO OF PAGES 129	7b. NO OF REFS 5
8a. CONTRACT OR GRANT NO DAHC15 67 C 0011 b. PROJECT NO Task T-57 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) P-728 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) None	

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Defense Advanced Research Projects Agency Arlington, Virginia 22209

13. ABSTRACT

The problem of extracting tracks of moving targets from sensor detections accompanied by a dense mixture of random false detections and recurrent false detections from stationary sources is examined. Algorithms for removal of fixed-target detections and for track establishment of moving targets are evaluated in terms of the tolerable false-alarm rate and minimum probability of detection for realizing a specified false-track-establishment rate and a specified probability of track establishment for moving targets, within a specified number of scans of the surveillance sensor. Some operational implications are briefly discussed in the form of a constrained resource allocation problem. Means for and benefits of multiple-sensor correlation are considered, and the problems introduced in attempting to provide surveillance of a mix of target types exhibiting significantly different kinematic characteristics are discussed.

DD FORM 1 NOV 65 1473

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Radar Track Establishment						
Sensor Data Processing						
Tactical Surveillance						

PAPER P-728

THE TRACK-ESTABLISHMENT PROCESS IN A MIXED FALSE-ALARM-RATE ENVIRONMENT

Robert D. Turner
Stanley Marder

June 1972

INSTITUTE FOR DEFENSE ANALYSES
SCIENCE AND TECHNOLOGY DIVISION
400 Army-Navy Drive, Arlington, Virginia 22202

Contract DAHC15 67 C 0011
Task T-57

PREFACE

The analysis documented by this paper is one of several undertaken in support of a study of the utility of a helicopter-borne radar for long-range surveillance of moving targets (personnel and vehicles) in conventional warfare. The study was performed under IDA Task T-57, and the overall results are presented in the following document:

> <u>ALARM System Performance Analysis</u>, IDA Study S-376, by Robert D. Turner, Arthur Krinitz, and Stanley Marder, December 1971.

The context of the study is that of a conceptual surveillance radar on a patrolling helicopter whose mission is to detect targets which are difficult to distinguish from the background of clutter echoes. Moreover, the application of the system output involves the commitment of resources (and possibly other actions) in response to apparent target detections. Accordingly, a major component of the study was the determination of means for converting low-quality single-scan detection data to high-quality tracks, recognizing that reactions to false tracks would both be costly in themselves and would dilute the application of resources to real targets.

The methods examined for achieving the high-quality system output goal were based on scan-to-scan correlation of the radar output. Correlation schemes of this general type are presently employed in existing operational systems, such as the Navy E-2 airborne early-warning and control aircraft. In general, however, the implementation of these correlation techniques has been based on the assumption of a far more benign natural interference (clutter) environment than is the case for the system concept that was studied. For example, the

E-2 automatic track-establishment equipment, used against aircraft over water, is designed to operate with a false-alarm rate (per scan) which is three orders of magnitude less than would be experienced by the study system concept. Other radars have, of course, been designed and deployed for overland surveillance of personnel traffic, but these radars operate from fixed platforms. For such systems, the problems of establishing tracks on real targets and suppressing false detections have generally been manageable by a human operator who can adapt to the interference scene, this scene being relatively invariant over many scans. By contrast, the study system concept would present the human operator with a continually changing background interference scene, even if the detection data from the radar are stabilized by conversion to a fixed coordinate system.

Accordingly, there was a serious question as to whether a human operator could deal with the problem, and there were no known existing automatic detection schemes which could be relied upon to do the job. In fact, there were no applicable theoretical results or experimental data for performance estimation, although a 1955 paper by N. Wax was partially relevant and provided considerable insight. The complications stemmed in part from:

1. The high single-trial false-alarm probabilities (1 to 2 percent) that must be tolerated in realizing even a mediocre single-scan probability of detection (0.8).

2. The presence, in the false alarms, of statistically recurrent false detections from fixed points in the area under surveillance; because of the motion of the radar platform, these false detections are unlikely to be as consistent from scan to scan as would be the case for a fixed radar.

3. The possibly meandering character of the scan-to-scan motion of real (personnel) targets.

4. The changing character of terrain and foliage masking, again due to the motion of the platform.

The analysis documented in this paper deals in varying detail with several kinds of automatic scan-to-scan correlation methods and was undertaken to assess the capabilities and feasibility of such methods.

The findings of the overall study with respect to the utility of the system concept were supported by analytically demonstrating the existence of realizable processing techniques, which can obtain useful track-establishment performance against real targets, while maintaining acceptably low false-track establishment rates. An important constraint on the utility of the system concept is exhibited in terms of the minimum number of scans which must be correlated to achieve a satisfactory balance between the real-target-track-establishment performance and the false-track-establishment rate. Because the scan rate of the radar is limited by the requirement for moving-target-detection processing, the rate at which the helicopter carrying the radar can patrol is constrained by the number of scans which must be correlated. The impact of this constraint on operational utility is discussed in the study referenced above. The impact of the trade-off between false-target rejection and real-target-track-establishment performance on the allocation of limited resources and the net worth of the system is examined in fairly abstract terms in this paper.

The problem of scan-to-scan correlation considered in this analysis is encountered in many applications of both active and passive sensors to surveillance missions. Accordingly, a conscious effort has been made to discuss qualitatively some aspects of the track-establishment problem which are beyond the immediate scope of the main study, and to delineate processing concepts with the potential for adaptation to a variety of military surveillance missions. Such missions include overland detection of aircraft, ocean surveillance, and tactical warning; and the systems employed for these missions may entail the use of sensor-to-sensor correlation as well as scan-to-scan correlation. The sensors themselves may provide nonkinematic data on a single-scan basis, or more complete kinematic data than the simple two-dimensional position report assumed in the analysis. The concepts that are exposed here--filtering to remove recurrent false detections, scan-to-scan integration to enhance real target detectability, and the development of track data for estimating the rate and direction of movement of real targets--are important for all of these situations. It is hoped that these concepts and the evaluation methods presented here will facilitate the realization of effective surveillance capabilities in the future.

ABSTRACT

The problem of extracting tracks of moving targets from sensor detections accompanied by a dense mixture of random false detections and recurrent false detections from stationary sources is examined. Algorithms for removal of fixed-target detections and for track establishment of moving targets are evaluated in terms of the tolerable false-alarm rate and minimum probability of detection for realizing a specified false-track-establishment rate and a specified probability of track establishment for moving targets, within a specified number of scans of the surveillance sensor. Some operational implications are briefly discussed in the form of a constrained resource allocation problem. Means for and benefits of multiple-sensor correlation are considered, and the problems introduced in attempting to provide surveillance of a mix of target types exhibiting significantly different kinematic characteristics are discussed.

CONTENTS

Preceding page blank

I. SUMMARY

The work presented here was undertaken in support of a study of the potential utility of a tactical surveillance system employing a helicopter-borne moving-target-detection radar. The results of that study are documented in a separate publication.* It was realized during the effort that the problem of track establishment is of interest for many system applications; for example, several papers were given which touched on this subject at the 15th AGARD Avionics Panel Symposium on Techniques for Data Handling in Tactical Systems (November 1968). Notwithstanding, there appears to be very little theoretical work published, beyond the 1955 paper of N. Wax (Ref. 1).

The particular problem considered here was that of establishing tracks for moving targets which are detected by a mobile scanning radar in a high-false-alarm-rate environment. The class of radars considered yields single-scan probabilities of detection in the range 0.7 to 0.9 with single-trial false-alarm probabilities in the range 0.005 to 0.02. Such a sensor can easily yield as many as 5000 false detections per scan. In general, the task of track establishment is to reject false detections and to provide high-confidence reports on real targets. By associating a number of detections of a real target, the track-establishment process discriminates against false detections and provides data on the position, rate, and direction of movement of real targets.

The basic problem is considered in two parts: suppression of temporally correlated false detections, which result from the existence

---

*R.D. Turner, A. Krinitz, and S. Marder, ALARM System Performance Analysis, IDA Study S-376, December 1971.

of discrete sources whose locations are fixed in the surveillance domain; and extraction of real-target tracks from the background of random false detections and residual temporally correlated false detections. It is shown that conceptually simple algorithms can be employed to obtain 90 to 95 percent suppression of the temporally correlated false detections; such techniques, however, impose stringent requirements on the quality of data stabilization and set a lower bound on minimum-detectable rates of target motion (apart from any limitations imposed by the surveillance sensor). Notwithstanding these constraints and the added processing burden imposed by the use of such techniques, distinct performance benefits result if the temporally correlated false detections represent more than 20 percent of the total false-detection input.

Two classes of real-target extraction procedures are considered: run tests, which require an uninterrupted sequence (from scan to scan) of detections for track establishment; and a somewhat more general class of recursive procedures which impose a greater processing burden. It is found that the difference in processing burden for the two classes is typically small compared to the total processing burden, and that the recursive techniques yield substantially better track-establishment performance than the run tests.

For the target and sensor parameters considered in this study, integration of 12 scans of detection data can yield a 0.85 probability of track establishment for real targets with average false-track establishment rates of one per minute, one per hour, and one per day, if the single-scan probability of detection exceeds 0.71, 0.82, and 0.88, respectively. If 18 scans are integrated, the required values for the single-scan probability of detection are 0.60, 0.69, and 0.74, respectively. The sensor false-detection parameters leading to these results are 5600 false detections per minute, of which 2400 are temporally correlated false detections, and 3200 are random false detections. Real targets are assumed to be constrained in maximum speed to the extent that the location of the target on a given scan is within a window containing 32 resolution cells centered on the location of the target on the previous scan. It is also assumed that the scan-to-scan motion of a real target is sufficiently correlated that the target location on a given scan is within a window containing nine resolution cells centered on a location predicted from the location data obtained from the two preceding detections of that target. The target is also assumed to move at least from one resolution cell to the next during a single scan period. Accordingly, the ratio of maximum to minimum target speed is 5:1 for surveillance with no prior information as the target heading, and 12:1 for surveillance along a known route. If greater variability requires a 50 percent increase in window size, achievement of the aforementioned track-establishment performance requires 6 to 8 percent increase in the single-scan probability of detection, or a 33 percent reduction in the false-detection rate.

An elementary model for determining the consequences of committing resources in response to the output of the surveillance system output is given. The model reflects the effects of resource constraints, as well as the real-target track-establishment performance and false-track-establishment rate of the surveillance system. It is shown in a numerical example that realizable track-establishment facilities can realize a net payoff which is 55 to 58 percent of that which would be obtained from an unrealizable "perfect" (no missed real targets, no false tracks) system. However, the track-establishment processing parameters must be fairly closely controlled, reflecting the relative losses for committing resources against false tracks, for not committing resources against real targets, and the payoff for committing resources against a real target.

A review is given of some problems of multiple-sensor correlation and of the advantages which can be realized from utilizing map data and other inputs indicating preferred target routes. Finally, a discussion is presented of the problems introduced by attempting surveillance of multiple target classes, which exhibit great differences in speed. It is shown that somewhat more complex track-establishment processing techniques may provide answers to the questions that are raised.

II. INTRODUCTION

A. BACKGROUND

The subject of this paper is the problem of extracting meaningful descriptions (tracks) of moving objects, using surveillance sensor data which are quite noisy, which is to say that they are obtained under conditions of relatively intense interference. As stated, the problem is quite general and very broad in scope, because of the variety of target types and interference phenomena; sensor mechanisms, parameters, and modes of operation; and surveillance domains.

Targets may vary in size (a parameter meaningful only in the context of the interference environment) from relatively small (personnel) to very large (ships). They may move on trajectories governed by physical laws or otherwise relatively predictable, which are somewhat random in character, or which are evasive in some sense. Their speeds may range from slow (personnel) to very fast (satellites and missiles), speed being a parameter which is meaningful only in the context of the sensor parameters and requirements placed on the tracking process.

The interference environment may arise from the intrinsic noise of the sensor itself, or it may stem from external sources, localized or distributed, in the surveillance domain. Such external sources may occur naturally, may arise from the sensing process itself (clutter), or may arise from intentional efforts to degrade the sensor performance.

The surveillance domain can be one-dimensional (targets moving along known paths), two-dimensional (targets moving on the surface of the earth), or three-dimensional (targets moving underwater, in the air or in space). It can also be argued that the dimensionality of

Preceding page blank

the surveillance domain is specified in part by the sensor characteristics; thus, a passive IR search set reports detections of objects moving in three-dimensional space in terms of a two-dimensional coordinate system. A pencil-beam coherent pulsed radar, on the other hand, may include range rate as well as range, azimuth, and elevation in its detection report, and it may be essential that the tracking process be viewed in the context of a surveillance domain of more than three dimensions.

The sensors themselves may be active or passive, in the latter instance relying on effects generated by the target itself, or external sources. The physical mechanism employed for conveying the existence of the target to the sensor may be seismic, acoustic, or electromagnetic in character, or, conceivably, a combination of these.

The detection system which provides inputs to the track-establishment and tracking process may consist of a single sensor, or of several sensors. The total surveillance domain may be observed simultaneously, or elements may be examined in time sequence. In the first instance, the observation process may be continuous or intermittent in character (sampling). The second instance is usually referred to as scanning. Some sensors employ combinations of these, e.g., sampling in one dimension of the surveillance domain, and scanning to cover the other dimensions.

In the employment of multiple sensors, the track-establishment and tracking problem is influenced by the degree of similarity of the sensors, and whether the coverages (coverage being that portion of the surveillance domain perceived by a sensor) of the sensors overlap, are contiguous, or disjoint (the last implying a requirement to interpolate or extrapolate the motion of the target between the coverage domains of the individual sensors).

The intent of the discussion just presented is to illustrate the manifold character of the surveillance process. To the authors' knowledge, there is no comprehensive theory of this process. The surveillance problems which motivated the study reported here involved a combination of sensor parameters, target characteristics, and interference environment which appeared to preclude the application of routine analytical methods, forcing the authors to acquire an admittedly embryonic understanding of the general problem. While it is believed that the study results have immediate utility for the applications which motivated the study, it is also hoped that they will illuminate part of the path toward a general theory and better understanding of the surveillance process.

B. GENERALITIES AND SPECIALIZATION

As an initial step in formulating the general surveillance problem, a model of the sensor system which supports the surveillance function will be described. An individual sensor provides a sequence $\{R_i\}$ of (apparent) detection descriptions, or reports each of which contains, at least in part, a partial kinematic descriptor. In general, $R_i = (K_i, C_i)$ where K_i is the kinematic description of the apparent target, and C_i contains nonkinematic information which describes the apparent target in other ways (size, color, irradiance, etc.). The report R_i is an indication that the sensor may have observed a target of interest at a location (or one of a set of possible locations) implied by the geometric component G_i of K_i, and which is moving on a trajectory (or one of a set of possible trajectories) implied by the dynamic component D_i of K_i. Thus, the kinematic component of the report can be written as $K_i = (G_i, D_i)$, and the entire report can be written as $R_i = (G_i, D_i; C_i)$.

Associated with each of these components of the report is a resolution cell which can be written $\Delta R_i = (\Delta G_i, \Delta D_i; \Delta C_i)$, the components of which are measures of the uncertainty in the true value of the component, given that the report corresponds to a real target.

The kinematic component of the report will be referred to as geometrically complete if it implies the location of the suspected target at a unique point in the surveillance domain, given known constraints on the trajectory of the target. This does not necessarily mean that the geometric component G_i provides such location explicitly. The kinematic component of the report will be referred to as dynamically complete if it (with known constraints) implies that the target is moving on a unique trajectory in the surveillance domain. In general, there may not exist a dynamically complete kinematic descriptor for a target. The simplest example of a kinematically complete descriptor would be three orthogonal Cartesian coordinates and the corresponding velocities of a particle moving in the absence of external forces.

The kinematic component of the report will be referred to as geometrically sufficient if the data (together with known constraints on the target motion) imply the location of the apparent target within the resolution cell ΔG that would be obtained with the next report on that target. This definition is somewhat vague, in that the terms "within" and "next report" have not been defined; it is hoped that the reader will be indulgent in such instances. There is an obvious extension of the concept of geometric sufficiency to kinematic sufficiency.

It is assumed that the density of real targets in the surveillance domain is such that the possibility of a composite report, based on the observation of two or more independent unresolved targets, can be ignored. (The notion of independence can be described in a negative sense; if a number of targets move along similar trajectories and are individually unresolved by the sensor, then they will appear to be a single target to the sensor. Such targets are not independent.)

In general, the objective of the tracking process is to extract a surveillance-oriented description of real targets of interest, while discarding false reports and reports on targets of no interest. An important phase of this process is the association of the reports resulting from multiple detections of a target. The manner in which this is accomplished hinges critically on the extent to which the reports are complete or sufficient, in the sense just described.

The first function of the association process is to establish the track of a real target in such a manner that false reports are unlikely to lead to (false) established tracks. When this is done, the target kinematic description available from the track may be more complete or precise than that available from an individual report. Having established the track, the second function of the association process is to associate subsequent reports on the target with the track; this operation may be facilitated by the improved target description available from the track history.

It will be noted in passing that there can be a dual interaction between the kinematic and nonkinematic components of the detection report. First, the nonkinematic components may facilitate the task of report association. Conversely, the aggregation (via track establishment) of the sequence of nonkinematic descriptors of a target may permit inference of a more precise nonkinematic description. The development of a track also assists in the association of reports from dissimilar sensors, thereby providing a more comprehensive nonkinematic description of a target than is available from a single sensor.

As was implied earlier, the report sequence from a sensor will generally contain false detections. The occurrences of false detections can manifest different kinds of correlation as the sensor repeatedly samples or scans its surveillance domain. One source of false detections is the self-noise of the sensor, and is generally independent from sample to sample or from scan to scan.

If a particular location in the surveillance domain persistently yields a higher false-detection rate than would be obtained from independent false detections alone, the false detections can be described as temporally correlated. This is meant to convey the notion that certain geometric resolution cells are not only likely to produce a significantly higher probability of multiple false reports over several scans than would be the case for independent false detections, but that the likelihood that one resolution cell exhibits this properly is independent of whether other resolution cells are so affected.

Another type of temporal correlation arises when there are interference sources that can cause time-related false detections from multiple sensors.

The notion of spatial correlation in false detection reports arises when the occurrence of a false report from one geometric resolution cell influences the likelihood of obtaining false reports from other (usually adjacent) cells. Finally, some sources of interference can exhibit both temporal and spatial correlation (noise jamming of radar being one example); the statistical description of such situations can become quite complex.

The specific problem considered in this paper deals with a scanning sensor which provides detection reports on apparent targets moving on the surface of the earth. The detection reports contain only geometric (apparent target location) data. The interference environment is assumed to result in a superposition of two kinds of false-detection sequences. The elements of one sequence are independent within a scan and from scan to scan, meaning that the occurrence of a report from a particular geometric resolution cell does not influence either the probability of obtaining false reports from other resolution cells, or the probability of obtaining a false report from the same resolution cells on subsequent scans. The elements of the second sequence are temporally correlated, and can be thought of as arising from a spatial distribution of discrete interference sources in the surveillance domain. Thus, the presence of such a source in one resolution cell is assumed not to affect the likelihood of there being such a source in other resolution cells, but does result in substantially higher probability of obtaining a false report on a scan-to-scan basis than would be the case for the independent source. Insofar as the reports obtained during a single scan are concerned, there is no distinction between a report stemming from the independent sequence, one arising from a fixed, discrete interference source, and a report resulting from detection of a real target.

With regard to the targets which are to be tracked, it is assumed that the real targets reported by the sensor will move at a speed which is equal to or greater than some minimum speed, and which is less than or equal to a maximum speed. It is further assumed that the real targets are moving purposefully, meaning that the path of a target is such that the target will eventually go from its present location to a definite destination. Thus, while the path may exhibit some meandering characteristics, it will also exhibit a preferred direction toward an objective; it is assumed that neither the direction nor the objective is known to the surveillance system beforehand. A final characteristic to be associated with real-target reports is that they will only be obtainable over a finite time period, which can be termed the target exposure time. This constraint on the surveillance process can arise for several reasons. First, the target may be masked from the sensor during portions of its excursion; thus, the exposure time is limited by the time the target is not masked. Second, the sensor itself may be moving, so that the target is within the field of view of the sensor for a finite time. Finally, it may be essential to detect the target at some time before it reaches its destination; the available exposure time will then be limited by the time required for the target to traverse from the perimeter of the surveillance domain to its objective, less the advance notice required.

The foregoing discussion presumes that whatever steps can be taken to discriminate between interference and manifestations of real targets on a single-observation basis have been taken. Because the sensor is assumed not to extract dynamic data at the time the observation is made, the information to be reported on an apparent target detection is the location. The task of the track-establishment process is therefore one of exploiting the data obtained from several scans to suppress the false reports and to associate the reports from real targets into tracks. In addition to providing a substantially higher confidence indication as to the existence of a target, the track may indicate the objective of the target. This indication of the objective would be inferred from the representation of the target path in the form of the associated location reports exhibited in time sequence. Conversely, such a representation of a false track may not exhibit a definite direction or trend of movement which would be associated with a purposeful excursion. Thus, the representation of a track affords a possible means for discriminating against false tracks; however, this possibility has not be explored in detail.

Some other features of the specific problem will be mentioned. First, the presence of temporally correlated interference and minimum target speed constrains the scanning rate for "simple" track-establishment schemes; reports of slowly moving real targets will not be distinguishable from the reports from fixed interference sources on the basis of successive scans unless the targets move from one resolution cell to another in the interim. The implied requirement (which is not mandatory, but simplifies both the analysis and realization of the track-establishment processor), together with the minimum target speed specification, sets an upper bound on the scanning rate. Taken with the finite exposure time, the scan-rate limit sets an upper bound on the number of observations of real targets that are available to establish their tracks.

A further complication arises from a stipulation that the probability of obtaining a report on a real target on a particular scan (i.e., the single-scan probability of detection) is significantly different from unity. For the purpose of analytical simplification, it is assumed that this probability does not change for a given target during its excursion through the surveillance domain. The stipulation arises from consideration of two factors. First, the target may be masked from the sensor at the moment that the sensor is examining the region (subset of the surveillance domain) in which the target is located. This source of degradation is assumed to be independent from scan to scan. The second factor results from the fact that it is frequently impossible to establish detection criteria in the sensor which simultaneously achieve good suppression of interference and yield a high probability of real-target detection. A compromise must therefore be adopted which realizes a useful balance between the rate of false detection reports and the probability of obtaining a report on a real target. In many instances, this compromise forces a lower probability of real-target detection than would otherwise be desired.

C. MOTIVATION AND OBJECTIVES

The problem which motivated the work reported here is that of extracting meaningful surveillance information from the target-detection reports generated by an MTI radar on an elevated platform. The PPI photographs of Fig. 1 provide an illustration of the difficulties involved. These show the output of a high-quality tower-mounted radar. Figure 1a shows the equivalent of unprocessed video; the display was generated by modulating the clutter return so that it would be passed by the Doppler processor. Figure 1b shows Doppler-processed video taken a few minutes later, and exhibits an atypically low blip count. Approximately 2 percent of the radar resolution cells have apparent targets in them.

The processed radar output can have contributions from the following sources:

(1) <u>Discrete (Fixed) Targets</u>

These result in apparent moving-target detections which repeatedly appear in the same resolution cells from scan to scan with a probability significantly higher than the fraction of cells which exhibit detections on a single scan. They may be due to strong, fixed scatters, but insofar as further processing is concerned the consistency of their appearance in particular resolution cells (temporal correlation) is their most significant characteristic.

*The authors are indebted to the Harry Diamond Laboratories for these pictures.

(a) Clutter Map

(b) Doppler-Processed Video

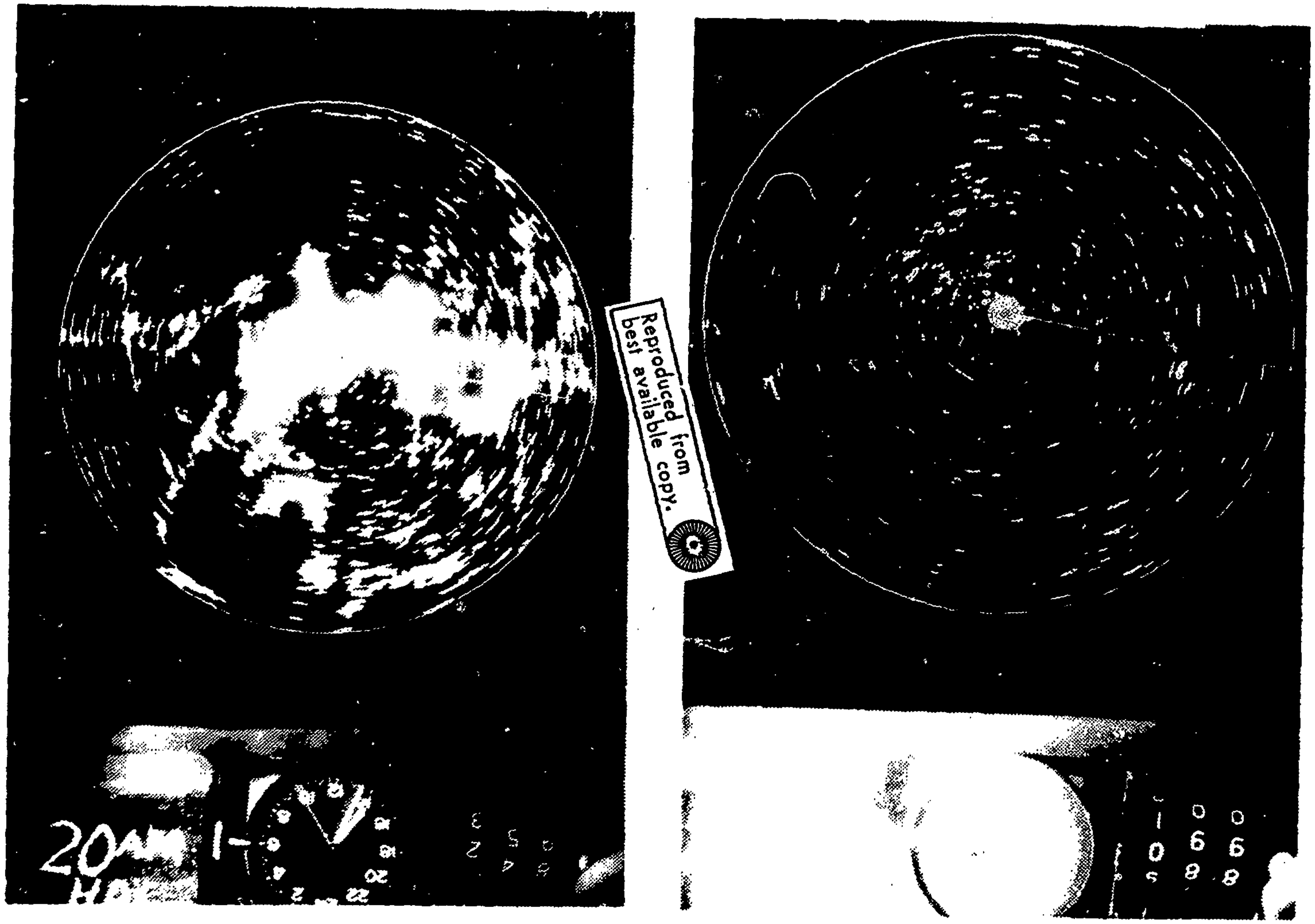

FIGURE 1. Real Clutter

(2) Random Clutter

The spatial correlation of detections stemming from MTI-processed clutter returns has not been established.* In this analysis, it is assumed that clutter detections not associated with discrete clutter are statistically independent from one resolution cell to another within a scan and from scan to scan.

(3) Receiver Noise

Assuming that good engineering design successfully minimizes the effects of nonrandom sources within the radar receiver, it appears to be reasonable to expect that false detections due to receiver noise are statistically independent within the scan and from scan to scan.

(4) Purposeful Motion

The direction of motion of targets of interest is normally correlated over several scans. It is this motion which the radar system is designed to detect.

In some MTI systems, the problem of discrete clutter has been attacked by a technique (discrete-target blanking) of desensitizing the radar for those resolution cells which exhibit returns much stronger than the average. It is assumed that even if this feature is included in the sensor, there will be a residual component which leads to false detections exhibiting the scan-to-scan correlation properties of discrete clutter.

*Dr. R. H. Fox has pointed out that enhancement of post-processing clutter levels due to wind may exhibit temporal-spatial correlation characteristics in the form of moving patches of higher levels. Clusters of "biological" clutter (birds, insects) may also exhibit spatial correlation.

The objectives of this paper, insofar as the motivating problem is concerned, are to:

(1) demonstrate the existence of track-establishment techniques which will permit the extraction of useful moving-target surveillance information under high false-detection rate conditions;

(2) establish contraints or requirements on the sensor operation which are imposed by the track-establishment function;

(3) delineate the effects of parameters of the track-establishment technique on its performance.

With regard to the first point, a typical value of the probability of obtaining a false detection from a single resolution cell on a single scan would be 0.02, with half of the false detections arising from discrete sources and the other half arising from random clutter and receiver noise. It is anticipated that under these conditions, a single-scan probability of detecting a real target in the range of 0.7 to 0.9 can be obtained. Thus, the processor will be exposed to about 2000 false detections per 100,000 resolution cells observed by the sensor, and can expect an average of 7 to 9 detection reports on a real target in 10 scans.

With regard to the second point, the primary constraint imposed (apart from requiring the lowest possible false-detection rate and the highest possible probability of detecting a real target) stems from the number of scans required to achieve satisfactory track-establishment performance. Given the scan-rate limitation mentioned earlier, this requirement implies a minimum target exposure time, and constrains the application of the system to regions sufficiently free of features which would mask the target, and constrains the rate of movement of a sensor on a moving platform (for given sensor coverage).

The performance of the track-establishment processing technique is specified in terms of three interdependent factors:

(1) the probability that real target tracks are detected,

(2) the rate at which false tracks are established, and

(3) the number of scans required to make a decision regarding the presence of a track.

The relative importance attached to these factors is strongly dependent upon the system application, and reflects the penalties associated with reacting to false tracks and failing to detect real targets.

III. TRACK-ESTABLISHMENT PROCESSING

A. CONCEPTS

The track-establishment processing techniques considered here exploit the purposeful movement of real targets of interest, in order to separate the detections of such targets from false detections. Thus, the only data* employed by the processor are the locations, and occurrence times associated with the apparent target detections. This approach is realistic, because in many practical situations, the location data constitute the only basis available for discrimination. Three conceptual schemes will now be discussed.

1. Time Compression

The time-compression technique involves the storage of apparent target detections from a number of scans; these are displayed to a human operator in rapid time sequence. Tracks are established on the basis of the operator's perception of apparently correlated sequences of detection. Success has been claimed for this technique in dealing with aircraft targets, which exhibit virtually rectilinear motion over several scan periods, in a false-detection environment containing a preponderance of discrete sources (temporally correlated clutter). It is not clear how well the technique would serve against targets which exhibit meandering trajectories, or in a false detection environment which gives rise to a significant number of false detections that are independent from scan to scan. The time-compression technique is relatively easy to implement, but was not considered further in this study, because:

*There are certain minor exceptions to this remark; in particular, the false detection rate estimated by the processor will be employed for control of the decision thresholds.

Preceding page blank

a. it does not appear to lend itself to the use of discrimination clues other than the location sequence alone;

b. it does not appear to have a potential for multiple-sensor data integration, particularly for dissimilar sensors or sensors which provide their reports asynchronously.

In the context of this study, these two reasons suggest that the time-compression technique has limited growth potential. Perhaps the most important reason for discarding the technique in this study is that it was not possible to determine quantitatively the performance available from using the technique, especially including the effects of operator fatigue and a changing interference environment. In many situations, a skilled and highly motivated operator who is familiar with the surveillance domain can sometimes detect moving targets under incredible conditions. (See Ref. 1 for a discussion of operator capabilities.) However, it is by no means obvious that the particular display technique employed has much effect on his performance.

2. <u>Spatial Templates</u>

A second technique is to use templates or spatial filters, which, figuratively speaking, consist of masks with apertures corresponding to possible target trajectories over a predetermined period of observation. The template is scanned over the stored detections from a number of scans. Track establishment is based on a determination that the number of apparent target detections falling within the aperture of the template exceeds a predetermined threshold, i.e., that the number is significantly greater than would be obtained from false detections alone.

The practical shortcoming of the template approach stems from the difficulty in synthesizing and using enough different apertures to account for different target trajectories. This number increases multiplicatively with the number of possible target trajectories over the period of observation. The technique appears not to be useful except for instances in which the target motion is severely constrained, e.g., moving along a known route or in a known direction. Another shortcoming of the technique stems from the fact that it does not take into account the time order in which the apparent target detections appear. This factor introduces a quite significant collapsing loss or, equivalently, a substantial increase in the false-track establishment rate, over that which would be obtained using a technique which takes the order of detections into account.

For these reasons, the template approach was not considered further in this study.

3. Recursive Techniques

A class of techniques can be envisioned in which the time-ordering and spatial filtering features of the two preceding techniques are combined. The template is, in effect, generated recursively, using time-ordered sequences of detections which have already been associated.

The track-establishment technique examined in this study is an extension of recursive scan-to-scan correlation methods that have been employed previously in a variety of applications, ranging from airborne early warning radar to seismic systems for infiltration detection. The basic concept was apparently first described in the open literature by Nelson Wax (Ref. 2), and has the presently incontestable advantage that it is both realizable in practical form and analytically tractable. Three factors distinguish the present effort from previous studies:

1. The false-detection rate that must be tolerated in the present context substantially exceeds (by two or more orders of magnitude) the false-detection rates which have been assumed in previous endeavors. This imposes a substantial burden for achieving an efficient processing scheme, both in a statistical sense and in the sense of hardware utilization.

2. The false detections are assumed to stem from two sources, one of which is temporally correlated (tending to produce recurrent false detections from discrete points in the surveillance domain), and the

other being independent from scan to scan and from point to point within a single scan. The burden for efficient processing suggests the necessity of exploiting the different statistical characteristics of the two sources. This point is further complicated by the fact that the sensor may be on a moving platform.

3. This factor arises from the characteristics assumed for the target trajectory. In earlier works, it has been customary to assume that the location of the target on the next scan can be inferred with good accuracy (e.g., within one or two resolution cells) from prior data.* In the present work, the target is permitted to meander to some extent, so that its location on the next scan can only be coarsely inferred from its location on the previous scan and its general direction and speed of movement.

Figure 2 presents a block diagram of the overall track-establishment processing concept. Incoming data from the sensor and its platform are stored in a buffer. The quantized radar data consist of a sequence of ranges at which apparent targets were detected. These are derived from the radar processor as an ordered sequence within a readout sweep; the readout sweep rate is determined by the radar antenna azimuthal resolution and the antenna scanning rate; thus, for a resolution of 0.5 degree and a scanning rate of 4 degrees/second (360 degrees in 90 seconds), a readout rate of the order of eight range sweeps per second would be used. Within each sweep, the radar processor reports the ranges at which apparent target detections occurred. For the present

*Target-trajectory smoothing algorithms which are consistent for constant-velocity targets or for constant-speed targets moving on trajectories with constant radii of curvature have been implemented.

context, the expected number of such reports per sweep is the single-trial false-detection probability multiplied by the number of range-resolution cells in the sweep. The platform data consist of the instantaneous platform position in the coordinate system employed by the processor, and the instantaneous antenna (azimuthal) pointing angle.

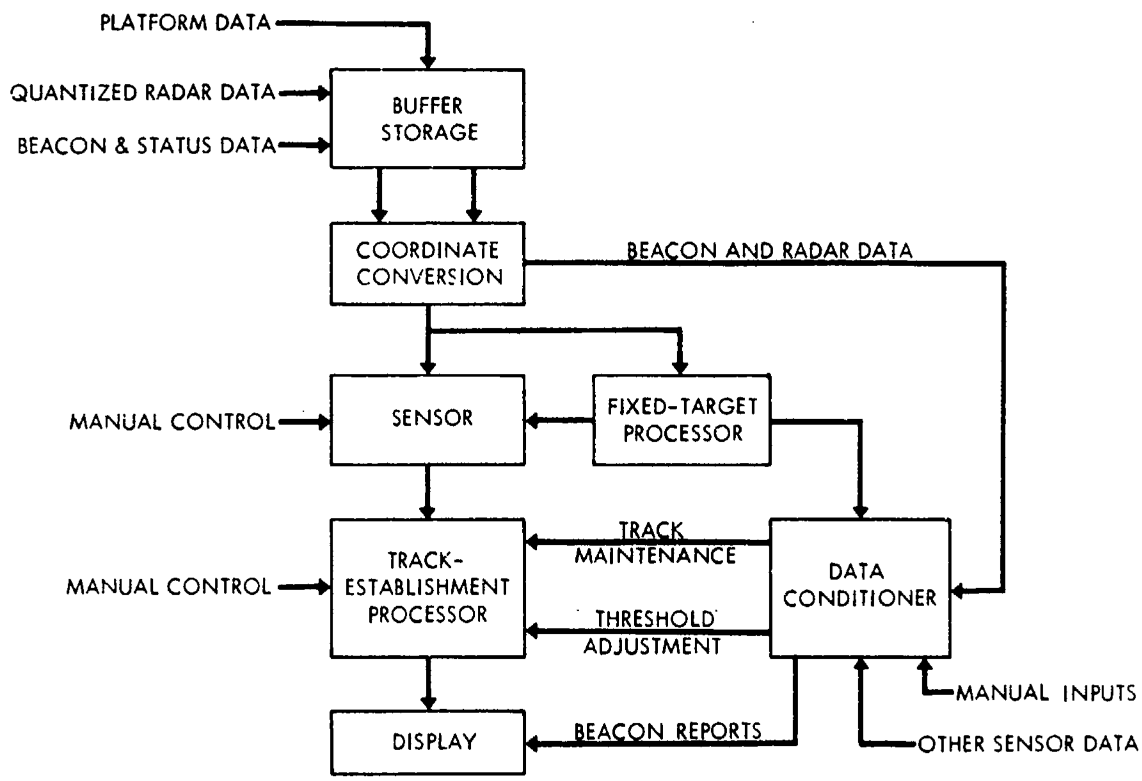

FIGURE 2. Track-Establishment Processor Concept

Also shown as an input are beacon and status data, which are in the same general format as the detection reports. The beacon data pertain to cooperative targets and are assumed to be essentially free of false reports; however, they must be associated with radar detections of the same targets. The status data input reflects the fact that the radar may inhibit detections in certain range bins (discrete target blanking), and may be capable of sensing portions of the surveillance region which are masked by terrain. The status data, therefore, provide the processor with an indication as to when not to expect detection reports.

The radar data obtained from buffer storage are converted to the fixed coordinate system employed by the system and are then sent to the fixed-target processor and to the track-establishment processor after being subjected to a censoring operation. The fixed-target processor identifies individual resolution cells or somewhat larger localized areas which exhibit persistent detections. The censoring operation serves to remove these detections from the input to the track-establishment processor. The output of the fixed-target processor is also sent to the data conditioner, to facilitate track maintenance when targets are moving through areas which contain fixed targets. A more detailed discussion of the fixed-target-removal process will be presented subsequently.

The track-establishment processor associates returns obtained on a number of scans, using a recursive algorithm which will be discussed below, to determine whether the detections exhibit the characteristics of purposeful movement. When the criteria for track establishment are met, the history of the returns forming the track are automatically displayed for the operator.

The data conditioner performs a variety of functions. As has already been noted, it can use fixed-target data to maintain track while a target passes through regions which have been censored by the fixed-target-removal process. It can use radar status data to determine terrain masking and high clutter conditions, and can modify the track-establishment processor to accommodate to these conditions. In addition, it can accept inputs from other sensors for processing and beacon signals and map data for direct display. Also, the data conditioner can estimate the false detection rate on the basis of detection reports which are not associated with eventual target tracks, thereby providing a basis for control of the track-establishment threshold.

Finally, the data conditioner permits the entry of detection reports from other sensors (which can be employed to assist or augment the track establishment process) and manual inputs which indicate regions of special interest (in which, for example, lower track-establishment thresholds might be employed).

Under normal conditions the history of target detection reports which led to track establishment are automatically displayed to the operator for verification as a purposefully moving target. In addition, the operator can interact with the processor in a variety of ways. He can enter sensor data and reference data manually. He can modify the parameters of the track-establishment processor and also the rules for holding tracks which passed the threshold for automatic establishment; he can modify the fixed-target processing; and he can vary the display mode.

In the following sections an expanded discussion of the fixed-target and track-establishment processing functions is given. The data conditioning functions are not as easily generalizable, and therefore require more specific definition to warrant detailed description. The buffer storage, coordinate conversion, and display functions are not sufficiently novel to require additional analysis in this paper.

B. FIXED-TARGET REMOVAL

The discussions presented here deal with the problem of false detections that persistently occur from fixed locations in the surveillance domain, i.e., temporally correlated false detections. A representative value for the fraction of the resolution cells seen by the sensor which exhibit such behavior would be 1 percent, which is comparable to the random (i.e., independent from scan to scan) false-detection rate. It will be seen subsequently that the false-track-establishment performance of the processor is quite sensitive to the total false-detection rate at the processor input. Thus, exploitation of the temporal correlation of fixed-target detections to reduce their rate to a small fraction of the random false-detection rate can result in a substantial reduction of the false-track-establishment rate, or an increase in the sensitivity of the processor for real-target detections.

Such detections can arise in an MTI radar from two sources: fixed targets which exhibit phase modulation, causing the fixed target return to be passed by the Doppler processor in the radar; and very strong fixed reradiators whose returns are modulated by the operation of the radar itself. One example of a target of the first kind would be a windmill; trees which librate under the influence of strong winds and clusters of insects or birds may also be a source of fixed-target detections. The second type of fixed-target detection results from targets such as buildings which give rise to abnormally large radar echoes; if such echoes are modulated by virtue of the radar scanning process, platform motion, or sources within the radar itself, then the side-bands introduced may be accepted by the Doppler processor and thereby produce false detections which persistently recur from scan to scan.

For a stationary radar the removal of fixed-target returns is straightforward. In principle, it need be done only once, since (by definition) fixed targets do not change their location. In practice, changes in the environment can cause variations in the locations which consistently yield false detections. The determination of fixed targets must then be repeated at a period appropriate to these changes. If this period is long compared to the time required for track-establishment processing, then the same processor can be time-shared between the two tasks, with track establishment processing foresworn during the times of fixed target removal. This would substantially reduce the digital processing capability required of the system.

1. Mobile Sensor Implications

If the sensor is on a moving platform, two kinds of complications for the fixed-target-removal process arise. First, the spatial distribution of fixed targets in the surveillance domain may change, because the target characteristics may be aspect-dependent. That is, the sensor may be subject to temporally correlated false detections when viewing the fixed target from some aspects, but not from others; in addition, the apparent location of the fixed target as seen by the sensor may change with aspect. For a radar, the first effect occurs when the target produces strong echoes because of a concavity which acts like a corner reflector; when the concavity is masked from the radar by other parts of the target, its back-scattering cross-section is diminished and the likelihood of temporally correlated false detections is reduced. The second effect arises when the fixed target has a large vertical extent; the multipath or range fold-over structure exhibited by such a target depends on aspect, and the apparent location of the target can shift, especially as the range from the radar to the target changes.

The second complication arises from the need to stabilize the apparent-target detection reports obtained from the sensor, that is, to convert the location data from the moving coordinate system of the sensor to the fixed coordinate system employed by the track-establishment processor. The coordinate conversion process depends on knowledge of the position of the sensor platform, and, for radar at least, the antenna pointing angle. For the purpose of fixed-target removal, it is desirable that the scan-to-scan location uncertainty of an apparent target (in the fixed coordinate system) due to the coordinate-conversion operation be less than the uncertainty due to the sensor resolution. If this requirement is not met, then fixed-target detection reports will appear to exhibit motion when displayed (figuratively or actually) in the track-establishment processor. For a high-resolution sensor, this requirement can be quite severe. For radar, the uncertainty in position of the sensor platform in the fixed coordinate system must be less than the range resolution of the radar, and the pointing angle of the radar antenna must be known with an accuracy which is better than its angular resolution.

In any event, the consequences of these factors are that fixed targets can give rise to recurrent false detections, but the occurrence of such a report on a given scan is not certain; and that the locations associated with such reports, as seen by the track-establishment processor, will wander. The first consequence suggests that an effective rule for removal of fixed-target detections will not demand a unity probability of obtaining a detection report; the second consequence sets a minimum value on the distance which a real target moves from scan to scan if its track is to be established in the fixed-target-removal context.

In what follows, it is assumed that the migrations of the apparent location of fixed targets can be confined to a single geometric resolution cell of the sensor. The implication of this assumption for real targets (for the class of track-establishment algorithms considered subsequently) is that a real target must move from scan to scan by a distance at least equal to the sensor resolution along the direction of movement. For an MTI radar that rejects returns that do not exhibit apparent range rates above a threshold, the scan-to-scan movement requirement can be satisfied by choosing the scan period large enough to ensure that a real target exhibiting the minimum-detectable range rate would move a distance at least equal to the radar range resolution.

2. Fixed-Target Removal Algorithms

The techniques considered in this study for the removal of temporally correlated false detections have the following general features. The locations associated with new detection reports are compared with previously established locations of apparent sources of temporally correlated false detections. If a new detection report falls within a suitably defined neghborhood of a fixed-target location, it is censored from the data stream employed for track-establishment *per se*, and is used to update the history associated with the fixed target with which it was correlated. Fixed targets are established in the fixed-target-removal processor when a sufficient sequence of new detection reports fall within a common neighborhood, and are dropped from the fixed-target-removal processor catalog when the location is no longer within the sensor field of view, or when an insufficient number of new detection reports have occurred to justify its continuance.

The issues which must be addressed in considering the fixed-target-removal process are:

a. when is the processor burden for fixed-target removal justified;
b. what criteria should be employed for maintaining apparent fixed targets in the catalog;
c. how should the dimensions of the association neighborhood or correlation window be selected.

With regard to the first question, it will be noted that presence of fixed-target false detections in the input to the track-establishment processor forces the use of more stringent track-establishment criteria, or the acceptance of higher false-track-establishment rates. In the first instance, the consequence is one of reducing the probability of establishing tracks on real targets, or diminishing the class of real targets for which a specified probability of track establishment can be achieved. A brief examination of results to be presented subsequently indicates that for typical parameter values of the track-establishment process, the effects of false detections due to fixed-target returns are ignorable if they are spatially uncorrelated and constitute less than 15 percent of the total false-detection input to the processor. Conversely, such false detections will require a significantly higher single-scan probability of detection for a specified track-establishment probability, or yield a significantly higher false-track-establishment rate, if they amount to more than 35 percent of the total false-detection input to the processor. For example, if

35 percent of the false detections can be removed by the fixed-target-removal process, the mean false-track-establishment rate can be reduced from one per hour to one per day for a given track-establishment threshold.

These remarks do not fully answer the first question, of course. A conclusive answer can only be obtained by examining typical false-detection statistics for which the origins of temporally correlated false detections are known or can be ascertained, and by consideration of the processor burden imposed if fixed-target removal appears to be warranted.

With regard to the second issue, specific algorithms must be considered. The algorithm which will be described is but one possible scheme. Each new detection whose location falls outside an existing correlation window* is stored in the fixed-target catalog as a tentative fixed target. Each subsequent detection which falls within the neighborhood associated with a tentative fixed target is used to update the catalog entry, but is also passed to the track-establishment processor, if that detection does not result in the tentative fixed-target entry being changed to an established fixed-target entry. Updating of the catalog entry means adjusting the count of the number of detections which have been associated with a tentative or established fixed-target entry, and recomputing the centroid of the location of that entry. A tentative fixed-target entry becomes established when the number of associated detections in the last n scans reaches a predetermined value k. An established fixed-target entry reverts to a tentative status when the number of associated detections falls below k. A tentative entry in the fixed-target catalog is dropped when the number of associated detections in the last n scans is zero.

*That is, the association neighborhood for an established or tentative fixed target, or a correlation window for a tentative or established track.

More precisely, a new detection is deleted from the input to the track-establishment processor if it falls within the fixed-target-removal correlation window of an entry in the fixed-target catalog, and that entry shows k - 1 or more associated detections in the n -1 previous scans. If the entry shows exactly k - 1 associated detections in the previous n - 1 scans, the entry now fulfills the criterion for an established fixed-target entry, and its status is changed. Conversely, if an established fixed-target entry shows exactly k associated detections in the previous n scans, including one such detection on the nth previous scan, then it reverts to the tentative status on the next scan if there is no new detection report that falls within the correlation window for that entry.

Accordingly, the probability that a new fixed-target detection is removed is just the probability that the fixed target had been detected at least k - 1 times during the previous n - 1 scans; denoting the single-scan probability of receiving a fixed-target detection report by P_{FTD}, the probability P_{FTR} that a new detection is removed is given by

$$P_{FTR} = \sum_{m=k-1}^{n-1} \binom{n-1}{m} P_{FTD}^{m} (1 - P_{FTD})^{n-m-1}$$

Note that this is the conditional probability of removal, given that a new detection has occurred.

The use of fixed-target-removal processing can lead to accidental desensitizations by random (i.e., not temporally correlated) false detections. Thus, if a real-target detection occurs in a fixed-target-removal correlation window, it will be deleted from the input to the track-establishment processor if the criterion for removal is met. The target detection will of course be lost if the target is in the same resolution cell as a source of fixed-target detections. In addition, however, the real-target detection report will be removed if there were k - 1 or more random false detections which resulted in a false fixed-target entry at the location of the target. The probability of this occurrence is given by an expression like that for P_{FTR}, with P_{FTD} replaced by P_{FD}, the probability of a random false detection within a given resolution cell on a single scan.

The choice of the parameters k and n depends on several factors, including the desired level of fixed-target removal, the probability of obtaining a fixed-target detection on a single scan, the acceptable level of desensitization which results from accidental establishment of fixed-target entries due to uncorrelated false detections, and the acceptable processing burden.

Table 1 shows some relevant statistics yielded by the process for different values of k and n. The column labeled Probability of Removal is the probability that a new fixed-target detection will be censored from the track-establishment processor input, assuming that the single-scan probability of obtaining such a detection is 0.75. The columns headed Number of Desensitized Cells give the expected number of resolution cells that will be desensitized by random false detections for the given random false-detection probability. In computing these values, it is assumed that the correlation window for fixed-target removal is identical with the sensor resolution cell, and that the sensor examines 10^6 resolution cells per scan.

TABLE 1. FIXED-TARGET-REMOVAL STATISTICS

Parameters		Probability of Removal	Number of Desensitized Cells		
k	n		$P_{FD} = 0.02$	$P_{FD} = 0.01$	$P_{FD} = 0.005$
2	2	0.75	20,000	10,000	5,000
2	3	0.9375	39,600	19,900	9,975
2	4	0.9844	58,808	29,701	14,926
3	3	0.5625	400	100	25
3	4	0.8437	1,184	298	75
3	5	0.9492	2,336	592	149
4	4	0.4219	8	1	---
4	5	0.7383	32	4	---
4	6	0.8965	78	10	1

Examining the results given in Table , it can be seen that the "simplest possible" rule (k = 2, n = 2) may be sufficiently good for some situations. Assuming that the fixed-target false-detection rate is less than or equal to the random false-detection rate, then the total false-detection rate after fixed-target removal will be larger than the random rate by a factor no greater than 1.25, and from 0.5 to 2 percent of the resolution cells will be desensitized. Any attempt to obtain higher efficiency removal of fixed targets by simply increasing n, however, leads to fairly large numbers of desensitized cells. For k = 3, n = 4 and k = 3, n = 5, better rejection of fixed targets is obtained, and the number of desensitized cells is less than 0.24 percent. Taking k = 3, n = 5, and assuming that the false-detection rate due to fixed-target returns is three times the random rate, the total false-detection rate entering the track-establishment processor would be 1.15 times the random false-detection rate. This would generally result in a dramatic improvement in track-establishment performance, because the overall false-detection rate would have been reduced by a factor of almost 3.5 by the fixed-target-removal process.

The consequence of increasing k and n to obtain better fixed-target removal without an unacceptable level of desensitization is an increased processor burden. For the case k = 2, n = 2, the fixed-target-removal processor must retain each detection report for one scan, but most of the random false detections will be dropped after the next scan. In the case k = 3, n = 5, on the other hand, random false detections must be retained for at least three scans; however, a large fraction will be dropped at this point. The fixed-target-removal processor burden using k = 3, n = 5 will be approximately four times that imposed by using k = 2, n = 2.

With regard to the third question raised earlier, the dimensions of the correlation window for fixed-target removal depend on the amount of scan-to-scan jitter in fixed-target detections seen by the processor after coordinate conversion. The assumption implied in the foregoing discussions and used in subsequent analyses is that this jitter can be held to the dimensions of a sensor resolution cell.

There are several consequences of using larger correlation windows for fixed-target removal to overcome jitter problems. First, the number of desensitized resolution cells will be increased for a given random false-detection rate. Next, the fixed-target-removal processor burden due to random false detections will be increased. Finally, and perhaps most important, the minimum moving-target speed which can result in track establishment will be increased.

These consequences can be offset to some extent by employing k = 3 or k = 4 in the fixed-target-removal process. For k = 3, the moving target can remain within the correlation window for two successive scans without being rejected by the fixed-target-removal processor, unless random false detections have effected or initiated the fixed-target-removal operation. In the case k = 3, n = 5, one false detection within the correlation neighborhood of a slowly moving real target on any of the preceding three scans will be sufficient to cause two successive detections of the target to be rejected by the fixed-target-removal process. Assuming a random false-detection probability of 0.01 and a correlation window of four (2 x 2) resolution cells, the probability of rejection is about 0.12. The probability of rejection is somewhat better for k = 4, n = 6, because two false detections in the preceding four scans are required for rejection. The corresponding probability of rejection obtained in this instance is 0.009.

3. Summary

Removal of temporally correlated false detections due to fixed-target returns is attractive for stationary sensors, because the acquisition of the catalog of fixed targets can be accomplished on a time-shared basis with surveillance operation. In the case of a sensor on a moving platform, the task of acquiring and using such a catalog is complicated by the problem of data stabilization, and the changing scene perceived by the sensor. Accordingly, the fixed-target-removal process must be accomplished concurrently with the track-establishment process, and may set a lower bound on the minimum-detectable target speed, independent of the sensor motion-detection capabilities.

In many instances, multiple-scan correlation techniques to establish a catalog of fixed targets are advantageous from the standpoint of surveillance performance. Use of such techniques can yield dramatic reductions in the false-detection rates seen by the track-establishment processor (with concomitant improvements in track-establishment performance) without introducing significant losses in sensitivity to random false detections.

C. TRACK ESTABLISHMENT

This section presents a description of a class of scan-to-scan correlation algorithms for use in track establishment per se. Relevant performance measures will be stated and applied, to illustrate the various trade-offs that can be made in synthesizing a practical system. It will be assumed that the false detections in the input to the track-establishment processor are spatially and temporally independent in a statistical sense, which means that the occurrence of a false detection in the surveillance region on one scan does not influence the probability of occurrence of a false detection at the same point on subsequent scans, nor the probability of occurrence of false detections at other points in the surveillance region on the same or subsequent scans. This assumption is at least approximately valid for the interference conditions assumed in Section II-B, if fixed-target-removal processing is employed, because the fixed-target-removal processor censors those inputs which exhibit temporal correlation.

The performance measures for track establishment which are considered were stated earlier and are restated here:

(1) The probability of track establishment that is obtained for real targets

(2) The rate at which false tracks are established by the processor

(3) The number of observations of the target required for track establishment.

These three measures interact; it will be seen that the trade-off between the real-target track-establishment probability and the false-track-establishment rate improves as the number of observations, or scans, used for track establishment is increased. Increasing the number of scans, however, increases the time required to establish a track, because the scanning period will be constrained by a number of factors. An increase in the total time of observation required for track establishment reduces the overall surveillance system capability in several ways. In some situations, the most important reduction may take the form of an increase in reaction time. The time required for track establishment also implies a time interval during which the target must be within the effective coverage or field of view of the surveillance sensor. Thus, the system may be limited to surveillance of regions which are sufficiently free of masking that the time required for the target to transit an unmasked region exceeds the required exposure time. Finally, if the sensor is on a moving platform, the required exposure time may limit the area search rate.

Another measure of system performance not examined in this study is tracking accuracy. For the class of targets considered here, the uncertainty in target location is an ill-defined function of the uncertainty in the last observation of the target position that was correlated with the target track, the elapsed time since the last observation, the target velocity and its variability, and the false detection rate.

The determination of the three performance measures depends on several factors: the sensor performance as modified by the various intermediate operations (readout, coordinate conversion, and fixed-target removal, for example), the character of the target motion, and the track-establishment processing parameters. The general character assumed for the target was discussed in Section II-B. The relevant processing parameters assumed for analytical purposes presume that the target will continue to move in approximately the same direction and at approximately the same speed during the time between the last observation and the next detection as it did during the preceding interval. The relative, or fractional, variability permitted by the parameters is less for higher speed targets than for those which move more slowly.

The sensor performance characteristics are stipulated* in the form of P_D, the single-scan probability of detection of a moving target; P_{FD}, the single-trial probability of a false detection from the sensor. (P_{FD} is effectively the expected number of false detections from the sensor per scan divided by N, the number of resolution cells seen by the sensor during a scan.) Both of these parameters are assumed to be measured at the input to the track-establishment processor in Fig. 2, i.e., after fixed-target removal. Other parameters which are important are the minimum and maximum number of sensor resolution elements which the target can move during a scan; these are functions of the minimum and maximum target speeds, the dimensions of the sensor resolution cells, and the scan period. It is assumed for the purpose of analysis that the target moves a distance from one scan to the next that is measurable (i.e., resolvable) by the sensor. In the parlance to be used, the target at least moves during the scan period from the resolution cell it occupied on the last observation to an adjacent resolution cell. It may move a greater distance, of course. This assumption is consistent with maintaining a capability to establish tracks on real targets while exploiting the advantages of fixed-target removal.

As a basis for comparing different processing options, nominal required values have been assumed for P_{TE}, the probability of track establishment for moving targets, and R_{FTE}, the average false-track establishment rate. These are

$$P_{TE} = 0.85$$

$$R_{FTE} = 1 \text{ per minute}$$
$$1 \text{ per hour}$$
$$1 \text{ per day}$$

The three values assumed for R_{FTE} are respectively consistent with:

*Appendix C presents a glossary of the symbols which are used in the text.

(1) Operator evaluation of computer-established tracks.

(2) Changes in the sensor mode of operation (for diagnosis of suspect tracks) based on computer-established tracks, where the change in mode causes a diversion of the sensor from its normal surveillance function.

(3) Alerting of affected parties or allocation of material resources (e.g., weapon designation) based on computer-established tracks.

Apart from these qualitative considerations, the values assumed are quite arbitrary.

1. Track-Establishment Algorithms

The class of track-establishment algorithms considered here involves the correlation of stored track histories with incoming detection reports. A track history is a collection of detection reports which have been associated and aggregated by the processor, together with a track status indicator, or quality number, which will be denoted by the symbol Q. A detection report consists of a location (the position of an apparent target on a particular observation) and the time or scan number at which the observation was obtained. The time or scan-number datum permits reconstruction of the track history on a display. The track history may be stored in two forms: an archival form, which contains as many of the detection reports as are deemed necessary for interpretation of the target motion (e.g., threat ordering and inference of intent), and an associative form, containing the most recent detection report (or reports) needed for the track-establishment procedure and track maintenance. The archival track history is maintained only for established tracks, and need not contain the track status indicator.

The associative track history is used to predict the location of the apparent target at the time it will next be observed by the sensor. This prediction is employed to establish a correlation window for that track; the dimensions of the correlation window reflect the uncertainty in target location on the next possible observation. It is convenient to describe the correlation window in units of the sensor resolution. Thus, for a radar sensor, the correlation window might subtend a domain in the surveillance region including three azimuth-resolution cells and three range-resolution cells. Such a window will be described as a 9-cell (3 azimuth x 3 range) window, even though the window may be described within the processor in different coordinates.

During a scan, the correlation windows corresponding to the current locations under surveillance are called out and compared with the detection reports being received. Each detection report whose location falls within a window is associated with the track history which generated that window, and the track status indicator is updated in a manner which will be described subsequently. If no detection reports fall within the correlation window for an established track, the correlation window location is recomputed for the next scan. If no detection reports fall within the correlation window for an initial track or tentative track (tracks which have not been established), the quality number is updated and the correlation window location is recomputed for the next scan if the track is not dropped.

Detection reports which fall outside of all existing correlation windows are stored as initial tracks and assigned the quality number Q_1. On the following scan, an initial correlation window of N_1 cells is generated for each initial track.

The quality-number updating process is the same for initial and tentative tracks: for each track, the quality number is increased by an amount Q_+ if a detection report is associated with the correlation window for the track on a particular scan. Conversely, the quality number is decreased by an amount Q_- if no association occurs on a given scan. If the quality number for an initial track attains the value Q_i, its status changes to that of a tentative track.

If the quality number for a tentative track attains the value Q_0, its status changes to that of an established track; the operator is then alerted. If the operator does not discard the computer-established track, it is continued, but the quality number is not changed from scan to scan. If the operator chooses to do so, he can assign a manual-verification status to the track. The manually verified status indicator can be employed in a variety of ways, including the assignment of a priority to manually verified tracks over the fixed-target-removal processor with respect to incoming detection reports.

Conversely, if the quality number for an initial or tentative track falls to zero, the track and all data associated with it are dropped from the track-establishment processor memory.

The distinction between initial and tentative tracks is that the values of the quality-number increments Q_+ and Q_- may be different, and the correlation windows for initial tracks are different from those for tentative and established tracks. The reasons for this distinction are twofold. First, an initial detection (of a possible target) contains no information as to the direction and rate of movement of that target; the dimensions of the initial correlation window must reflect that fact. The track histories associated with tentative and established tracks, on the other hand, permit an inference as to the approximate rate and direction of movement of the targets, so that the dimensions of the correlation window for tentative and established tracks can generally be made smaller than the correlation window for initial tracks. Thus, the initial-track status is one which is only maintained until it is possible to estimate the approximate target speed and direction, i.e., only until a detection report is obtained in the initial correlation window.

In general, it is necessary to stipulate an upper bound on the speed of real targets for the purpose of sizing the initial correlation window. The radar sensor considered here is assumed to have an azimuthal resolution which subtends a distance five times the radar range resolution, and it is assumed that the maximum target speed is equivalent to five range-resolution cells per scan. Allowing for motion in any direction, the initial correlation window would be centered on the location of the initial detection, and would subtend 11 range-resolution cells and 3 azimuth-resolution cells. The center cell can be ignored, because the fixed-target-removal process will tend to suppress repeated detection reports from the same location. Thus, the initial correlation window contains $N_1 = 32$ resolution cells; this value was employed for the computations which will be reported subsequently.

The probability that at least one detection will fall within a given initial correlation window is

$$P_1 = 1 - (1 - P_{FD})^{N_1} \tag{1}$$

or

$$P_1 \cong N_1 \, P_{FD}$$

if this quantity is small compared to unity. It can be seen that the probability of dropping a false initial track is large only if the product $N_1 \, P_{FD}$ is small. It is therefore desirable to minimize the dimensions of the initial correlation window.

Letting N denote the total number of resolution cells examined by the sensor during a single scan, the expected number of false detections per scan will be NP_{FD}. Suppose that a fraction F of these generate initial tracks because they fall outside of all existing correlation windows. A lower bound on the expected number of correlation windows called out per scan is then just $NP_{FD}F$. The expected number of new false detections which fall in existing correlation windows is then at least $NP_{FD}F \, P_1$. Under steady-state conditions, however, the expected number of false detections is $(1 - F) \, NP_{FD}$. Thus,

$$(1 - F) \, NP_{FD} \geq NP_{FD}F \, P_1$$

which has the solution

$$F \leq 1/(1 + P_1) \tag{2}$$

Using this simple and conservative approximation, the expected number of false initial tracks created per scan that will have at least one false detection in their initial correlation windows on the next scan is

$$N_{FT} = NP_{FD}P_1/(1 + P_1) \quad (3)$$

It is to be noted that if $N = 10^6$, $P_{FD} = 0.01$, and $N_1 = 32$, then the number of false detections per scan is 10^4 and P_1 is 0.275. The expected number of false initial tracks with false detections in their initial correlation windows is 2157.

With these numbers in mind, it is possible to reach a conclusion regarding the step from initial track to tentative track. For the conditions stated, each scan results in 10,000 false detections and 7843 new false initial tracks. The next scan results in false detections corroborating 2157 of these, 5686 which are not corroborated, and 7843 new false initial tracks. The most obvious method for limiting the false-track population is to discard those initial tracks which are not corroborated on the next scan.

In the context of the general framework, this procedure is equivalent to setting $Q_- = Q_1$ for initial tracks, so that Q falls to zero if there is no detection within the correlation window for an initial track on the next scan.

Conversely, if there is a detection within the initial correlation window, the initial and subsequent detections provide an approximate estimate of the apparent target direction and rate of movement. Thus, the corroborated initial track can be declared a tentative track, which amounts to setting $Q_+ = Q_i - Q_1$; the dimensions of the correlation windows used subsequently (referred to hereafter as subsequent correlation windows) can be reduced in size (relative to the initial correlation windows), reflecting the knowledge of the apparent target direction and rate of movement.

The subsequent correlation windows for a tentative (or established) track are centered on the target location predicted for the next scan.

In this study, it is assumed that the size of the subsequent correlation window is held constant, comprising N_2 resolution cells (or the equivalent of N_2 statistically independent trials) of the surveillance sensor output. The prediction of the apparent target location for the next scan may be based on a simple extrapolation of the apparent target locations obtained from the last two detections, or more elaborate prediction rules may be employed.

The probability of at least one false detection appearing within a subsequent correlation window is

$$P_2 = 1 - (1 - P_{FD})^{N_2} \qquad (4)$$

For $P_{FD} = 0.01$ and $N_2 = 9$, $P_2 = 0.0865$. Thus, of the 2157 false tentative tracks declared per scan in the example given above, an average number of 187 will be (falsely) further corroborated on the next scan. If the criterion for continuation of a newly declared tentative track requires a correlating detection on the next scan following tentative track declaration, the reduction in false tentative tracks is indeed dramatic. However, the question must be raised as to what effect such a criterion has on track establishment performance for real targets.

In the next section, two classes of track-establishment algorithms will be analyzed. The first class considered is the run test, for which the track-establishment criterion is R correlated detections on R successive scans.* The advantage of the run-test class is that the tests reject false tentative tracks at the highest possible rate, and thereby minimize the processor burden. These tests are obtained by setting

$$Q_1 = 1 \qquad Q_- = Q$$

$$Q_+ = 1 \qquad Q_2 = R$$

*The run test is superficially similar to the time-compression technique mentioned in Section III-A.

The false-track establishment rate is controlled by adjusting R. The fact that $Q_i = 2$ for the run tests is immaterial.

In the second class of algorithms, denoted simply as Q tests, the value of Q_{-} is fixed at $Q_{-} = 1$, and Q_i is adjusted to allow continuance of the tentative track, even if correlating detections do not occur on one or more scans immediately following tentative track declaration. The false-track establishment rate is controlled by adjustment of the threshold Q_0, and the objective of the Q test is to trade increased processor burden for better real-target track-establishment performance.

2. Multiple Correlations

In the preceding discussions, the tacit assumption was made regarding the occurrence of multiple detections within a single correlation window. This assumption is that if two or more detections occur within a single correlation window, they will be aggregated and treated as a single detection, with the apparent target location being taken as the centroid of the locations corresponding to the several detections. The position errors which result can be accommodated by use of larger subsequent correlation windows when multiple detections occur, the size being adjusted according to the uncertainty in the predicted target position. Alternatively, the multiple detections can be treated as evidence of multiple targets, with subsequent correlation windows being established for each detection appearing within a correlation window. This matter is more important for initial tracks than for tentatively declared tracks. For the example given above, the average number of false detections appearing within initial correlation windows is 2510. Thus, if multiple detections are treated as multiple targets, there will be about 16 percent more false tentative-track declarations than if the multiple detections are aggregated. With aggregation, approximately 16 percent of the newly declared false tentative tracks will require larger subsequent correlation windows. Approximately 4 percent of the normal-sized ($N_2 = 9$) subsequent correlation windows will have multiple false detections.

The possibility will be mentioned of employing a hybrid procedure for dealing with the matter of multiple correlation. For this procedure, the processor aggregates or splits the apparent target locations, using larger subsequent correlation windows for aggregated detection reports and normal subsequent correlation windows for detection reports which are not aggregated. The allocation would be made so as to minimize the total number of resolution cells resulting from the procedure.

IV. PERFORMANCE ANALYSIS

A. INTRODUCTION

This section presents analytical results on the performance and processing burden associated with run tests and a broader class of recursive procedures, referred to here as Q tests, for track establishment. A glossary of the principal symbols used in the text is given in Appendix C. The performance parameters analyzed are P_{TE}, the probability of track establishment for a real target (characterized by a single-scan probability of detection P_D) and R_{FTE}, the average rate of occurrence of false established tracks, due to random false detections, characterized by a single-trial false-detection probability P_{FD}.

For the Q tests, it is assumed that tentative tracks are discarded after S scans. In the case of a patrolling sensor, the discarding process may take place implicitly, if the coverage of the moving sensor shifts to the extent that detection reports are no longer obtained from the region in which the apparent target is located. The impact of this assumption on the run tests is simply that the target must be detected on R successive scans during a total observation period of S scans.

In general, the analytical expression for P_{TE} is a polynomial of order S in P_D. It is assumed that real targets are within the field of view of the surveillance sensor for at least S scans.

The average false-track-establishment rate is given by

$$R_{FTE} = R_{FTTD} \quad P(TE|TTD) \tag{5}$$

where R_{FTTD} denotes the average rate of false tentative track declarations, and P(TE|TTD) is the probability of false-track establishment,

Preceding page blank

given false tentative track declaration. An approximate expression for R_{FTE} is

$$R_{FTE} = R_{FD}\, P_1/(1 + P_1) \tag{6}$$

where

$$R_{FD} = NP_{FD}/T \tag{7}$$

is the average false detection rate, and P_1 is expressed in terms of N_1 and P_{FD} by Eq. (1). The quantity $P(TE|TTD)$ is, in general, a polynomial of order S-2 in P_2, where P_2 is given in terms of N_2 and P_{FD} by Eq. (4).

The parameters P_D, P_{FD}, N, and T, are parameters of the surveillance sensor. In particular, P_D and P_{FD} are not completely controllable, although some trade-off (increasing P_{FD} to improve P_D) may be available within the sensor configuration. In fact P_D is likely to depend on the target characteristics, and will be unknown a priori at least to the extent that the target characteristics are not known beforehand. The value of P_{FD} can be estimated, however, and this estimate provides a basis for controlling R_{FTE}, via the choice of R for the run tests and via the choice of Q_o for the Q tests. The control process depends on N_1 and to an even greater extent on N_2, which depend both on the surveillance sensor parameters and the characteristics of target motion. The correlation-window dimensions, which determine N_1 and N_2, are selected within the track-establishment processor to obtain a proper balance between achieving a high probability of associating real targets, on the one hand, and a manageable level for R_{FTTD} and an acceptable value for R_{FTE}, on the other. This facet of the track-establishment problem is discussed further in Sections VI-B and VI-C.

In any event, it is assumed that the process is controlled so as to maintain R_{FTE} at a specified level. The assumed objective of the process design is to minimize the value of P_D required to achieve a specified value of P_{TE} (nominally 0.85), subject to the constraint on R_{FTE}.

Comparisons of the various tests for track establishment can be made in terms of the polynomial expressions for P_{TE} and $P(TE|TTD)$. For the sake of concreteness, numerical results will be presented for the following surveillance sensor parameters:

$N = 480{,}000$ (resolution cells per scan)
$T = 90$ sec (scan period

The values $N_1 = 32$ and $N_2 = 9$ will be assumed for this purpose.

B. ANALYTICAL RESULTS FOR THE RUN TESTS

1. Probability of Track Establishment

As was noted earlier, the probability of track establishment for real targets, using a run-of-R test, is just the probability of R or more successive, associated detections of the target in S ($\geq R$) scans. This probability can be computed analytically, using the recursion formula (Refs. 2 and 3)

$$P_{TE}(s + 1, R) = [1 - P_{TE}(s - R, R)]\, P_D^R\, (1 - P_D) + P_{TE}(s, R) \quad (8)$$

along with the initial conditions

$$\begin{aligned} P_{TE}(s, R) &= 0 \qquad s < R \\ P_{TE}(R, R) &= P_D^R \end{aligned} \quad (9)$$

The expressions for P_{TE} obtained by means of the above technique can be written in the form of a polynomial

$$P_{TE} = \sum_{k=R}^{S} C_k(R, S)\, P_D^k \quad (10)$$

Table 2 gives the polynomial expressions for P_{TE} for $R = 5$ and $S = 5$ to 18. Table 3 gives the polynomials for $R = 6$, $S = 6$ to 18, and $R = 7$, $S = 7$ to 18. Equation (10) can be cast in an alternative

TABLE 2. P_{TE} FOR RUN-OF-5 TRACK ESTABLISHMENT RULE

Number of Scans	P_{TE}
5	P^5
6	$2P^5 - P^6$
7	$3P^5 - 2P^6$
8	$4P^5 - 3P^6$
9	$5P^5 - 4P^6$
10	$6P^5 - 5P^6$
11	$7P^5 - 6P^6 - P^{10} + P^{11}$
12	$8P^5 - 7P^6 - 3P^{10} + 4P^{11} - P^{12}$
13	$9P^5 - 8P^6 - 6P^{10} + 9P^{11} - 3P^{12}$
14	$10P^5 - 9P^6 - 10P^{10} + 16P^{11} - 6P^{12}$
15	$11P^5 - 10P^6 - 15P^{10} + 25P^{11} - 10P^{12}$
16	$12P^5 - 11P^6 - 21P^{10} + 36P^{11} - 15P^{12}$
17	$13P^5 - 12P^6 - 28P^{10} + 49P^{11} - 21P^{12} + P^{15} - 2P^{16} + P^{17}$
18	$14P^5 - 13P^6 - 36P^{10} + 64P^{11} - 28P^{12} + 4P^{15} - 9P^{16} + 6P^{17} - P^{18}$

TABLE 3. P_{TE} FOR RUN-OF-6 AND RUN-OF-7 TRACK ESTABLISHMENT RULES

Number of Scans	P_{TE} for Run-of-6 Test	
6	P^6	----
7	$2P^6 - P^7$	P^7
8	$3P^6 - 2P^7$	$2P^7 - P^8$
9	$4P^6 - 3P^7$	$3P^7 - 2P^8$
10	$5P^6 - 4P^7$	$4P^7 - 3P^8$
11	$6P^6 - 5P^7$	$5P^7 - 4P^8$
12	$7P^6 - 6P^7$	$6P^7 - 5P^8$
13	$8P^6 - 7P^7 - P^{12} + P^{13}$	$7P^7 - 6P^8$
14	$9P^6 - 8P^7 - 3P^{12} + 4P^{13} - P^{14}$	$8P^7 - 7P^8$
15	$10P^6 - 9P^7 - 6P^{12} + 9P^{13} - 3P^{14}$	$9P^7 - 8P^8 - P^{14} + P^{15}$
16	$11P^6 - 10P^7 - 10P^{12} + 16P^{13} - 6P^{14}$	$10P^7 - 9P^8 - 3P^{14} + 4P^{15} - P^{16}$
17	$12P^6 - 11P^7 - 15P^{12} + 25P^{13} - 10P^{14}$	$11P^7 - 10P^8 - 6P^{14} + 9P^{15} - 3P^{16}$
18	$13P^6 - 12P^7 - 21P^{12} + 36P^{13} - 15P^{14}$	$12P^7 - 11P^8 - 10P^{14} + 16P^{15} - 6P^{16}$

form

$$P_{TE} = \sum_{m=R}^{S} C_m^* (R, S) P_D^m (1 - P_D)^{S-m} \quad (11)$$

where

$$C_m^* (R, S) = \sum_{k=R}^{m} C_k (R, S) \binom{S-k}{m-k} \quad (12)$$

in which case C_m^* (R, S) represents the number of ways that a run of R can be obtained in S scans with m detections. The coefficients given by Eq. (12) will be used for purposes of comparison with analytical results for the Q test.

Table 4 gives values of PTE for R = 5, 6, and 7, various values of S, and P_D = 0.7, 0.8, and 0.9. It can be seen, as might be expected, that the run tests yield good performance only for fairly high values of P_D.

TABLE 4. P_{TE} FOR RUN TESTS

R	S	P_D		
		.7	.8	.9
5	6	.2185	.3932	.6495
	9	.3698	.5898	.8267
	12	.5015	.7392	.9306
	15	.6045	.8327	.9718
	18	.6860	.8921	.9879
6	6	.1176	.2621	.5314
	9	.2235	.4194	.6909
	12	.3294	.5767	.8503
	15	.4191	.6845	.9165
	18	.4976	.7676	.9573
7	9	.1318	.2936	.5740
	12	.2059	.4194	.7174
	15	.2780	.5365	.8381
	18	.3423	.6253	.8992

2. False-Track Establishment Rate

The expression for $P(TE|TTD)$ for the run tests is simply

$$P(TE|TTD) = P_2^{R-2} \tag{13}$$

and is independent of S. This expression can be cast into a form appropriate for comparison with analytical results for the Q tests by noting that

$$P_2^{R-2} = \sum_{m=R-2}^{S-2} \binom{S-R}{m-R+2} P_2^m (1 - P_2)^{S-2-m} \tag{14}$$

The coefficient of $P_2^m (1 - P_2)^{S-2-m}$ in Eq. (14) is simply the number of ways that a run of R - 2 detections can be obtained (immediately following the 2 detections required for tentative track declaration), given m detections in S - 2 scans.

Table 5 gives values of the false-track establishment rate as a function of P_{FD} for R = 5, 6, and 7. It can be seen that for values of P_{FD} less than 0.012, the false-track establishment rate drops very rapidly when R is increased. This characteristic is, in one sense, an inherent weakness of the run tests; the single parameter does not provide great latitude in tuning the processing to the needs of the situation.

TABLE 5: R_{FTE} FOR RUN TESTS

P_{FD}	R=5	R=6	R=7
.001	.01 per day	---	---
.002	.33 " "	.01 per day	---
.005	1.06 per hour	1.13 " "	.05 per day
.008	9.63 " "	.67 per hour	1.12 " "
.010	.45 per min	2.32 " "	.20 per hour
.012	1.02 per min	6.28 per hour	.65 per hour
.014	2.02 " "	.24 per min	1.72 " "
.016	3.63 " "	.49 " "	3.98 " "
.018	6.04 " "	.91 " "	.14 per min
.020	9.49 " "	1.58 " "	.26 " "

Suppose for example, that the system is being operated so as to maintain the average false-track establishment rate below a nominal level of 1 per hour. At $P_{FD} = 0.008$, the average rate for $R = 6$ is 0.67 per hour, which is comfortably below the nominal level. However, if P_{FD} increases to 0.01, then R must be changed to 7, dropping the false-track establishment rate to 0.2 per hour, an unnecessarily low value. The impact of such a change can be seen in Table 4; with $P_D = 0.9$, and using 12 scans for track establishment, the probability of track establishment drops from 0.85 to 0.72, when R is changed from 6 to 7.

3. Processor Burden

The burden for track-establishment processing can be estimated as follows. Suppose that the scans of the surveillance sensor are arbitrarily indexed with index s. At the end of scan s, the processor will have received an average number, n_I, of new false detections, for which initial correlation windows will be generated on scan s + 1. The approximate expression for n_I is

$$n_I \cong \frac{NP_{FD}}{1+P_1} \tag{15}$$

The approximation, it will be recalled, is in the factor $F \cong 1/(1 + P_1)$, and is the same throughout all the discussions of processor burden. The value of n_I is the same for both run tests and Q tests.

In addition, there will be an average number n_{-1} of new false tentative tracks declared during scans (for which new subsequent correlation windows will be generated on scan s + 1) and arising from new false detections which occurred on scan s - 1. The approximate expression for n_{-1} is

$$n_{-1} \cong \frac{NP_{FD}P_1}{1+P_1} \tag{16}$$

Next, there will be an average number n_{-k} false tentative tracks originally stemming from new false detections which occurred on scan s - k:

$$n_{-k} \cong \frac{NP_{FD}P_1}{1+P_1} P_2^{k-1} \quad k = 2, \ldots, R - 1 \tag{17}$$

The total processor burden for generating subsequent correlation windows on scan s + 1 is obtained by combining Eqs. (16) and (17):

$$n_S = n_{-1} + n_{-2} + \cdots n_{-R+1} \cong \frac{NP_{FD}P_1}{1+P_1} \left(\frac{1-P_2^{R-1}}{1-P_2} \right) \tag{18}$$

The term P_2^{R-1} in Eq. (18) is ordinarily quite negligible, and a reasonably accurate upper bound for the total processor burden for generating subsequent correlation windows is

$$n_S < \frac{NP_{FD}P_1}{(1+P_1)(1-P_2)} \tag{19}$$

The ratio of the run-test processor burden for generating subsequent correlation windows to the burden for generating initial correlation windows is, comparing Eqs. (15) and (19), simply $P_1/(1 - P_2)$. (Note that neither n_I nor n_S includes the burden for fixed-target removal processing, display generation, and other functions.) For $N_1 = 32$, $N_2 = 9$, and $P_{FD} = 0.01$, the run-test processor burden for generating subsequent correlation windows is about 30 percent of that for generating initial correlation windows.

4. Performance Summary

Table 6 summarizes the track-establishment processing performance with run tests, in terms of the minimum values of P_D required to attain $P_{TE} = 0.85$, and the maximum values of P_{FD} for which the three nominal false-track-establishment rates can be maintained. For R = 5, reasonable values of P_D are required if 15 or more scans are available for track establishment, but low false-track-establishment rates are unattainable except with rather low values of P_{FD}. At R = 7, significantly higher values of P_{FD} can be tolerated while maintaining false-track-establishment rates at the lower levels, but the realization of satisfactory track-establishment performance on real targets requires values of P_D in excess of 0.87, or more than 18 scans.

TABLE 6. RUN TEST PERFORMANCE SUMMARY

	No. of Scans	Detection Criterion		
		R = 5	R = 6	R = 7
Minimum values of P_D to obtain $P_{TE} = 0.85$	S = 6	.9606	.9733	---
	S = 9	.9106	.9515	.9686
	S = 12	.8520	.8999	.9423
	S = 15	.8090	.8663	.9044
	S = 18	.7753	.8353	.8786
Maximum values of P_{FD} for				
R_{FTE} = 1/min		.01195	.01832	.02499
R_{FTE} = 1/hour		.00494	.00859	.01285
R_{FTE} = 1/day		.00255	.00490	.00786

C. ANALYTICAL RESULTS FOR THE Q-TESTS

1. Parameter Selection

The parameters available for processor control with the Q-tests are Q_i, Q_+, Q_-, and Q_0. The process of track establishment (or rejection) can be regarded as a random walk in the Q-domain, with steps occurring on a scan-to-scan basis. After tentative-track declaration, the walk starts at $Q = Q_i$, with a step upward of length Q_+ or a step downward, of length Q_-, after each scan. The values $Q = 0$ and $Q = Q_o$ are lower and upper absorbing barriers, respectively. As was noted earlier, all the parameters are assumed to be held constant for processing a given track, and Q_i is assumed to be held constant for all tracks. The value $Q_- = 1$ will be assumed; it is not apparent that this assumption entails any loss of generality.

The value of Q_i then determines how many scans can elapse, without subsequent correlation of a new tentative track, before that track is dropped. A new tentative track will be retained in the processor for $Q_i - 1$ scans without subsequent correlations. Thus, for $Q_i = 1$, the tentative track will be dropped on after the very next scan, if no detection occurs within the subsequent correlation window on that scan. Taking into account the possibility of an accidental subsequent correlation*, the probability of dropping a new tentative track of a real target is $(1 - P_2)(1 - P_D)$. For $P_{FD} = 0.01$, $N_2 = 9$, and $P_D = 0.7$, this probability is 0.274. The problem with choosing $Q_i = 1$ is, therefore, that there is an uncomfortably large probability that tentative tracks for real targets will be dropped**. For $Q_1 = 2$, the probability that the real-target tentative track is dropped after the two scans following tentative track declaration is 0.075, which is still roughly comparable to the nominally acceptable probability ($1 - P_{TE} = 0.15$) of not establishing track on a real target. For $Q_1 = 3$, the probability that the tentative track is dropped directly (i.e., after three scans following tentative-track declaration) is less than 0.021, which is probably

*Accidental correlations are due to false detections, and are not particularly helpful for track establishment on real targets.

**It is possible, of course, to retain dropped tentative-track declarations in the processor memory for use via a look-back algorithm, in the event a new tentative track is declared. This alternative was not examined.

suitably small, compared to the postulated required maximum value of $1 - P_{TE}$. For $Q_i = 4$, the probability that the tentative track is dropped after four scans following tentative-track declaration is less than 0.006, which is probably unnecessarily small.

The value $Q_i = 3$ was adopted for all analyses of the Q-tests on the basis of the rather subjective analyses just given. The impact of the choice of Q_i on the processor burden will be discussed later.

Polynomial expressions have been derived for P_{TE} and $P(TE|TTD)$ by means of the computational procedures described in Appendix A. Table 7 gives the polynomial coefficients for $Q_i = 3$, $Q_+ = 2$, and $S = 12$, for $Q_o = 9$ to $Q_o = 17$. The upper set of coefficients in Table 7 are the values of b_n in the representation

$$P(TE|TTD) = \sum_{n=3}^{10} b_n \, P_2^n \, (1 - P_2)^{10-n} \tag{20}$$

and the lower set of coefficients are the values of a_n in the representation

$$P_{TE} = \sum_{n=5}^{12} a_n \, P_D^n \, (1 - P_D)^{12-n} \tag{21}$$

Thus, the values of b_n represent the number of ways that the value Q_o can be equalled or exceeded during the ten scans (for $S = 12$) following tentative track declaration, given n false subsequent correlations. The values of a_n represent the number of ways that a track on a real target can be established ($Q \geq Q_o$ during 12 scans) given n correlated detections. For example,

$$P(TE|TTD) = 8 \, P_2^7 \, (1 - P_2)^3 + 45 \, P_2^8 \, (1 - P_2)^2 + 10 \, P_2^9 \, (1 - P_2) + P_2^{10}$$

and

$$P_{TE} = 25 \, P_D^9 \, (1 - P_2)^3 + 65 \, P_D^{10} \, (1 - P_D)^2 = 12 \, P_D^{11} \, (1 - P_D) + P_D^{12}$$

for $Q_o = 16$.

TABLE 7. POLYNOMIAL COEFFICIENTS FOR $Q_+ = 2$, 12 SCANS

Q_0	n=2	3	4	5	6	7	8	9	10	11	12
9	0	1	19	138	203	119	45	10	1		
10	0	0	5	65	203	119	45	10	1		
11	0	0	1	24	203	119	45	10	1		
12	0	0	0	6	89	119	45	10	1		
13			0	1	30	119	45	10	1		
14			0	0	7	119	45	10	1		
15			0	0	1	37	45	10	1		
16			0	0	0	8	45	10	1		
17			0	0	0	1	45	10	1		
9			0	8	112	486	491	220	66	12	1
10			0	0	31	263	474	220	66	12	1
11			0	0	7	112	421	220	66	12	1
12			0	0	0	31	248	219	66	12	1
13					0	6	105	214	66	12	1
14					0	0	29	188	66	12	1
15					0	0	5	88	66	12	1
16					0	0	0	25	65	12	1
17					0	0	0	4	58	12	1

Tables 8 and 9 present the corresponding polynomial coefficients for $Q_1 = 3$, $Q_+ = 4$; and $Q_1 = 3$, $Q_+ = 6$, respectively, for $S = 12$.

TABLE 8. POLYNOMIAL COEFFICIENTS FOR $Q_+ = 4$, 12 SCANS

Q_o	n=3	4	5	6	7	8	9	10	11	12
15	1	67	231	203	119	45	10	1		
16	0	34	231	203	119	45	10	1		
17	0	15	231	203	119	45	10	1		
18	0	5	231	203	119	45	10	1		
19	0	1	121	203	119	45	10	1		
20		0	55	203	119	45	10	1		
21		0	21	203	119	45	10	1		
22		0	6	203	119	45	10	1		
23		0	1	203	119	45	10	1		
24		0	0	83	119	45	10	1		
25				28	119	45	10	1		
26				7	119	45	10	1		
27				1	119	45	10	1		
28				0	119	45	10	1		
29				0	36	45	10	1		
30					8	45	10	1		
31					1	45	10	1		
32					0	45	10	1		
33					0	45	10	1		
34					0	9	10	1		
15		0	8	274	663	491	220	66	12	1
16		0	0	157	606	484	220	66	12	1
17		0	0	81	549	484	220	66	12	1
18		0	0	31	489	474	220	66	12	1
19		0	0	7	339	454	220	66	12	1
20				0	193	441	219	66	12	1
21				0	91	407	219	66	12	1
22				0	31	362	219	66	12	1
23				0	6	332	214	66	12	1
24				0	0	202	209	66	12	1
25						92	209	66	12	1
26						29	188	66	12	1
27						5	170	66	12	1
28						0	164	66	12	1
29						0	81	65	12	1
30							25	65	12	1
31							4	58	12	1
32							0	55	12	1
33							0	55	12	1
34							0	19	12	1

TABLE 9. POLYNOMIAL COEFFICIENTS FOR $Q_+ = 6$, 12 SCANS

Q_o	n=3	4	5	6	7	8	9	10	11	12
21	1	175	231	203	119	45	10	1		
22	0	111	231	203	119	45	10	1		
23	0	65	231	203	119	45	10	1		
24	0	34	231	203	119	45	10	1		
25		15	231	203	119	45	10	1		
26		5	231	203	119	45	10	1		
27		1	231	203	119	45	10	1		
28		0	231	203	119	45	10	1		
29		0	120	203	119	45	10	1		
30		0	55	203	119	45	10	1		
31		0	21	203	119	45	10	1		
32		0	6	203	119	45	10	1		
33		0	1	203	119	45	10	1		
34		0	0	203	119	45	10	1		
35			0	203	119	45	10	1		
36			0	83	119	45	10	1		
37			0	28	119	45	10	1		
38			0	7	119	45	10	1		
39			0	1	119	45	10	1		
21		0	8	427	663	491	220	66	12	1
22		0	0	342	637	484	220	66	12	1
23		0	0	250	637	484	220	66	12	1
24		0	0	-157	606	484	220	66	12	1
25			0	81	549	484	220	66	12	1
26			0	31	489	474	220	66	12	1
27			0	7	449	454	220	66	12	1
28			0	0	434	441	219	66	12	1
29				0	323	441	219	66	12	1
30					193	441	219	66	12	1
31					91	407	219	66	12	1
32					31	362	219	66	12	1
33					6	332	214	66	12	1
34					0	322	209	66	12	1
35					0	322	209	66	12	1
36					0	202	209	66	12	1
37					0	92	209	66	12	1
38					0	29	188	66	12	1
39					0	5	170	66	12	1

Comparison of the entries in Tables 7, 8, and 9 reveals certain equivalences; thus $Q_+ = 6$, $Q_o = 38$ will result in identical performance, because the polynomial expressions for P_{TE} and $P(TE|TTD)$ are identical. Three equivalences (2, 12; 4, 26), (2, 16; 4, 30), and (2, 17; 4, 31) between $Q_+ = 2$ and $Q_+ = 4$ are shown in Tables 7 and 8. Tables 8 and 9 show 1[illegible] equivalences between $Q_+ = 4$ and $Q_+ = 6$, ranging from (4, 16; 6, 24) to (r, 27; 6, 39). In a sense, therefore, there will be greater differences in performance between $Q_+ = 2$ and $Q_+ = 4$ than between $Q_+ = 4$ and $Q_+ = 6$.

The choice of Q_+ determines the degree to which Q_o can be adjusted to realize a false-track establishment rate which is at or just below a stipulated tolerable level. The degree of adjustment is determined by the manner in which the polynomial coefficients in $P(TE|TTD)$ decay with increasing Q_o. It will be noted in the tables of the coefficients for $P(TE|TTD)$ that the <u>n</u>th coefficient is unity when $nQ_+ = Q_o - Q_i$, which is simply an expression of the fact that the only way track establishment can occur with this number of false detections is by a run of n false detections following tentative-track declaration. The value of n increases by unity when Q_o is increased by Q_i, and there are consequently $Q_i - 1$ intermediate steps between these major shifts in the pattern of coefficients.

A perhaps important secondary benefit of employing a higher value of Q_+ is that greater latitude is available when combining multisensor data of unequal quality. Adopting $Q_+ = 4$ for the best input (in the sense of minimum false detection rate) permits $Q_+ = 1$, 2, or 3 to be used for detection reports from lower quality inputs. The choice $Q_+ = 6$ gives even greater latitude in this regard. It will be seen, however, that higher values of Q_+ result in increased processor burden. Accordingly, the value $Q_+ = 4$ was taken for computation of the polynomial coefficients for $S = 18$, the results of which are given in Table 10.

TABLE 10. POLYNOMIAL COEFFICIENTS FOR $Q_+ = 4$, 18 SCANS

Q	n=3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
15	1	73	1243	6124	9640	11559	10722	7722	4290	1807	559	120	16	1		
16	0	34	799	6124	9640	11559	10722	7722	4290	1807	559	120	16	1		
17	0	15	481	6124	9640	11559	10722	7722	4290	1807	559	120	16	1		
18	0	5	261	4274	9640	11559	10722	7722	4290	1807	559	120	16	1		
19	0	1	127	2774	9640	11559	10722	7722	4290	1807	559	120	16	1		
20		0	55	1668	9640	11559	10722	7722	4290	1807	559	120	16	1		
21		0	21	924	9640	11559	10722	7722	4290	1807	559	120	16	1		
22		0	6	454	9640	11559	10722	7722	4290	1807	559	120	16	1		
23		0	1	239	5830	11559	10722	7722	4290	1807	559	120	16	1		
24		0	0	83	3265	11559	10722	7722	4290	1807	559	120	16	1		
25				28	1680	11559	10722	7722	4290	1807	559	120	16	1		
26				7	784	11559	10722	7722	4290	1807	559	120	16	1		
27				1	327	11559	10722	7722	4290	1807	559	120	16	1		
28				0	119	6056	10722	7722	4290	1807	559	120	16	1		
29				0	36	2910	10722	7722	4290	1807	559	120	16	1		
30					8	1266	10722	7722	4290	1807	559	120	16	1		
31					1	490	10722	7722	4290	1807	559	120	16	1		
15		0	14	754	8616	30159	44853	43395	31820	18564	8568	3060	816	153	18	1
16		0	0	361	5315	27186	43626	43181	31807	18564	8568	3060	816	153	18	1
17		0	0	171	3831	24309	42339	43010	31807	18564	8568	306	816	153	18	1
18		0	0	61	2211	19609	40349	42540	31797	18564	8568	3060	816	153	18	1
19		0	0	13	1137	14417	38705	42110	31753	18564	8568	3060	816	153	18	1
20				0	523	9710	35061	41407	31698	18563	8568	3060	816	153	18	1
21				0	217	5386	31571	40625	31654	18563	8568	3060	816	153	18	1
22				0	57	3296	28046	39400	[illegible]	18563	8568	3060	816	153	18	1
23				0	12	1616	21361	37680	31334	18558	8569	3060	816	153	18	1
24				0	0	700	14388	35309	31119	18547	8568	3060	816	153	18	1
25						260	8643	33004	30734	[illegible]	8568	3060	816	153	18	1
26						71	4597	29539	30251	[illegible]	8568	3060	816	153	18	1
27						11	2156	26491	29488	18472	8568	3060	816	153	18	1
28						0	878	18843	28350	18435	8567	3060	816	153	18	1
29						0	297	[illegible]	27280	18352	8567	3060	816	153	18	1
30							73	5953	24364	18156	8567	3060	816	153	18	1
31							10	2695	22377	17995	8560	3060	816	153	18	1

2. Performance Results

Table 11 presents values of the probability of track establishment obtained with processing over 12 scans, as a function of Q_o for $Q_+ = 4$, $P_D = 0.7$, 0.8, and 0.9. Table 12 gives the same results for processing over 18 scans. Table 13 presents values of the average number of false established tracks per hour, as a function of Q_o, that are obtained with various values of P_{FD} and with processing over 12 scans. Table 14 gives corresponding values of the average false-track establishment rate for processing over 18 scans. Comparison of the values obtained for R_{FTE} with $S = 12$ with those obtained with $S = 18$ reveals that the Q_o used with $S = 18$ must be higher by 1 to 3 to achieve approximately equivalent false-track establishment performance. However, the improvement in real-target track establishment performance in going from $S = 12$ to $S = 18$ more than offsets the effect of increasing Q_o. Thus, for $S = 12$, $Q_o = 24$ yields approximately the same false-track performance as for $Q_o = 27$ with $S = 18$. Despite the increase in Q_o, however, the value of P_{TE} obtained (referring to Tables 11 and 12) increases from 0.57 to 0.86 for $P_D = 0.7$, and from 0.84 to 0.99 for $P_D = 0.8$.

Tables 15 through 18 give the maximum tolerable values of P_{FD} to realize stipulated false-track-establishment rates and the values of P_D required to achieve a real-target track-establishment probability of 0.85. Comparison of the results for S - 12 (Tables 15, 16, 17) indicates that no choice of Q_+ yields clearly superior performance. However, the increases in the required values of P_D for a unity increase in Q_o are somewhat smaller for the larger values of Q_+. Thus, the processor can adapt to changes in P_{FD} more efficiently if $Q_+ = 4$ or 6 than if $Q_+ = 2$. The advantages realized, however, are only important if small changes in the required value of P_D are significant.

Comparing the results of Table 16 ($S = 12$) with those of Table 18 ($S = 18$) again reveals the dramatic reduction in the required values of P_D obtained with the longer processing period. Conversely, the values of P_{FD} which can be tolerated with $S = 18$ are roughly double those which can be tolerated with $S = 12$.

TABLE 11. P_{TE} FOR Q TEST: $Q_+ = 4$, S = 12

	P_D		
Q_o	.7	.8	.9
15	.8683	.9755	.9988
16	.8533	.9678	.9982
17	.8353	.9627	.9979
18	.8144	.9551	.9971
19	.7730	.9393	.9955
20	.7360	.9248	.9939
21	.6997	.9089	.9919
22	.6667	.8927	.9897
23	.6422	.8777	.9864
24	.5749	.8370	.9788
25	.5235	.8075	.9741
2[illegible]	.4850	.7680	.9632
27	.4404	.7422	.9552
28	.4290	.7301	.9492
29	.3385	.6410	.9170
30	.2775	.5809	.8953
31	.2368	.5283	.8628
32	.2249	.5111	.8508
33	.2249	.5111	.8508
34	.1333	.3565	.7253

TABLE 12. P_{TE} FOR Q TEST: $Q_+ = 4$, $S = 18$

	P_D		
Q_o	.7	.8	.9
15	.9456	.9993	1.0000
16	.9428	.9991	1.0000
17	.9402	.9989	1.0000
18	.9360	.9985	1.0000
19	.9317	.9981	1.0000
20	.9256	.9975	1.0000
21	.9200	.9970	1.0000
22	.9136	.9962	1.0000
23	.9038	.9950	1.0000
24	.8939	.9937	.9999
25	.8827	.9921	.9999
26	.8713	.9903	.9999
27	.8608	.9882	.9998
28	.8418	.9846	.9997
29	.8251	.9809	.9997
30	.8042	.9760	.9995
31	.7885	.9714	.9994

TABLE 13. AVERAGE FALSE-TRACK ESTABLISHMENT RATE (per hour) for $Q_+ = 4$, $S = 12$

Q_o	P_{FD}								
	.002	.005	.008	.010	.012	.014	.016	.018	.020
15	.03	3.52	42.13	134.46					
16	.01	1.57	21.97	77.81	---				
17	---	.89	13.70	52.23	145.28				
18		.53	9.35	38.76	112.54	---			
19		.25	4.94	19.36	58.11	145.01			
20		.11	2.35	9.60	30.04	78.04	---		
21		.05	1.24	5.27	17.26	47.01	130.79		
22		.03	.75	3.36	11.62	33.32	85.18	---	
23		.02	.59	2.72	9.75	28.76	69.97	148.86	---
24		.01	.24	1.14	4.19	13.03	30.87	67.59	137.24
25		---	.10	.48	1.82	6.24	14.34	32.00	67.74
26			.04	.23	.92	3.65	8.02	18.42	41.20
27			.03	.15	.66	2.91	6.22	14.54	33.61
28			.02	.14	.62	2.79	5.92	13.90	32.35
29			.01	.05	.20	.92	2.02	4.87	11.44
30			---	.01	.07	.29	.71	1.83	4.38
31				.01	.03	.13	.38	1.06	2.62
32				---	.03	.10	.33	.96	2.36
33					.03	.10	.33	.96	2.36
34					.01	.02	.08	.22	.55

TABLE 14. AVERAGE FALSE-TRACK ESTABLISHMENT RATE (per hour) for $Q_+ = 4$, [illegible] = 18

Q_o	P_{FD} .002	.005	.008	.010	.012	.014	.016	.018	.020
15	.03	4.38	50.56	---					
16	.01	2.25	30.84	133.42					
17	---	1.36	20.89	95.71					
18		.98	12.32	59.30	---				
19		.36	7.10	36.17	117.58				
20		.19	3.14	22.55	76.93				
21		.10	2.61	14.96	53.55	---			
22		.06	1.75	10.71	40.28	132.56			
23		.03	.95	6.10	23.74	71.85	---		
24		.01	.52	3.49	14.10	44.15	115.88		
25		.01	.29	2.08	8.78	28.61	77.81		
26		---	.17	1.35	6.00	20.41	57.56	---	
27			.13	1.01	4.68	16.50	47.82	119.40	---
28			.06	.53	2.49	9.01	26.71	68.96	154.10
29			.03	.27	1.35	5.04	15.42	40.45	93.98
30			.02	.15	.79	3.09	9.82	26.66	63.86
31			.01	.10	.54	2.22	7.30	20.42	50.17

TABLE 15. Q-TEST PERFORMANCE SUMMARY ($Q_+ = 2$, S = 12)

Q_o	P_{FD} for R_{FTE} = 1/Day	P_{FD} for R_{FTE} = 1/Hour	P_{FD} for R_{FTE} = 1/Minute	P_D for P_{TE} = 0.85
9	.00242	.00452	.01000	.71172
10	.00358	.00612	.01237	.73906
11	.00441	.00733	.01419	.76094
12	.00569	.00905	.01678	.79375
13	.00684	.01067	.01928	.81406
14	.00806	.01214	.02109	.83281
15	.00956	.01430	.02459	.86719
16	.01137	.01667	.02797	.88281
17	.01241	.01786	.02934	.89531

TABLE 16. Q-TEST PERFORMANCE SUMMARY ($Q_+ = 4$, S = 12)

Q_o	P_{FD} for			P_D for
	R_{FTE} = 1/Day	R_{FTE} = 1/Hour	R_{FTE} = 1/Minute	P_{TE} = 0.85
15	.00219	.00394	.00853	.6820
16	.00266	.00459	.00953	.6984
17	.00297	.00506	.01025	.7078
18	.00328	.00547	.01206	.7383
20	.00434	.00703	.01344	.7523
21	.00486	.00778	.01456	.7656
22	.00534	.00841	.01537	.7766
23	.00562	.00869	.01569	.7852
24	.00637	.00981	.01766	.8062
25	.00722	.01106	.01966	.8180
26	.00806	.01212	.02109	.8328
27	.00852	.01266	.02166	.8414
28	.00862	.01275	.02172	.8461
29	.00987	.01462	.02497	.8703
30	.01137	.01666	.02797	.8828
31	.01241	.01784	.02934	.8953

TABLE 17. Q-TEST PERFORMANCE SUMMARY ($Q_+ = 6$, $S = 12$)

Q_o	P_{FD} for R_{FTE} = 1/Day	P_{FD} for R_{FTE} = 1/Hour	P_{FD} for R_{FTE} = 1/Minute	P_D for R_{TE} = 0.85
21	.00195	.00346	.00741	.67188
22	.00220	.00384	.00809	.68359
23	.00241	.00418	.00878	.68906
24	.00276	.00460	.00953	.69844
25	.00292	.00504	.01025	.70781
26	.00328	.00546	.01078	.71875
27	.00348	.00570	.01103	.72813
28	.00355	.00576	.01109	.73438
29	.00397	.00634	.01219	.74297
30	.00434	.00702	.01344	.75234
31	.00486	.00777	.01456	.76563
32	.00534	.00840	.01537	.77656
33	.00562	.00868	.01569	.78516
34	.00569	.00875	.01575	.78984
35	.00569	.00876	.01575	.78984
36	.00637	.00982	.01766	.80625
37	.00722	.01104	.01966	.81797
38	.00806	.01213	.02109	.83281
39	.00851	.01266	.02166	.84141

TABLE 18. Q-TEST PERFORMANCE SUMMARY ($Q_+ = 4$, S = 18)

Q_o	P_{FD} for			P_D for
	R_{FTE} = 1/Day	R_{FTE} = 1/Hour	R_{FTE} = 1/Minute	P_{TE} = 0.85
15	.00215	.00381	.00797	.55547
16	.00256	.00434	.00869	.56563
17	.00284	.00473	.00922	.57422
18	.00320	.00523	.01000	.58516
19	.00361	.00578	.01078	.59609
20	.00402	.00633	.01156	.60625
21	.00439	.00681	.01219	.61563
22	.00475	.00723	.01269	.62422
23	.00520	.00786	.01366	.63594
24	.00567	.00848	.01459	.64688
25	.00614	.00911	.01547	.65625
26	.00658	.00963	.01609	.66563
27	.00691	.00997	.01647	.67344
28	.00748	.01078	.01772	.68516
29	.00809	.01159	.01891	.69531
30	.00867	.01231	.01984	.70625
31	.00911	.01281	.02041	.71406

Finally, it will be noted that the results generally reflect the value of fixed-target-removal processing if the frequency of occurrence of such detections is comparable to that for random false detections. In Table 16, for example, doubling the value of P_{FD} forces an increase in the required value of P_D by 10 to 18 percent. A similar effect is found in Table 18, as well.

3. Processor Burden

A cumbersome analysis along the lines of the processor burden calculation for the run tests indicates that an upper bound on the burden for generating subsequent correlation windows (for the Q tests) is

$$n_S < \frac{Q_i \; NP_{FD} \; P_1}{(1+P_1) \; [1-(Q_+ +1) \; P_2]} \qquad (22)$$

This expression, which is analogous to Eq. (19), is somewhat imprecise for larger values of Q_+, and of course does not reflect the fact that many false tentative tracks will be discarded after S scans. Indeed, a more precise expression, for $Q_i = 3$, $Q_+ = 4$, and $S = 12$ is

$$n_S < \frac{3 \; NP_{FD} \; P_1}{1+P_1} \; [1 + 5 \; P_2 + 25 \; P_2^2] \qquad (23)$$

For S = 18, the bound

$$n_S < \frac{3 \; NP_{FD} \; P_1}{1+P_1} \; [1 + 5 \; P_2 + 25 \; P_2^2 + 125 \; P_2^3] \qquad (24)$$

is obtained. The discrepancy between Eq. (22) and Eqs. (23) and (24) is most important for larger values of P_2. Thus, for $N_2 = 9$, $P_{FD} = 0.02$, the value $P_2 = 0.16625$ is obtained. The factor $1/[1-(Q_+ +1)P_2]$ of Eq. (22) is then 5.0 for $Q_+ = 4$, but the bracketed quantities in Eqs. (23) and (24) are 2.5 and 3.1, respectively. However, the discrepancy is less than 10 percent for $P_{FD} = 0.01$.

4. Comparison With the Run Tests

A number of specific comparisons have been made between the Q-test performance results and those for the run tests. Table 19 gives a comparison of the polynomial coefficients which may provide some analytical insight regarding the differences. The run-test coefficients have been cast into the form of the Q-test coefficients, using Eqs. (12) and (14). The Q-test coefficients that are shown are for values of Q_o which yield the same leading terms in the polynomial expressions. Table 19 also lists* the binomial coefficients (which are upper bounds on the possible coefficient values) for purposes of comparison. The coefficients listed are for S = 12, Q_i = 3, and are tabulated under the number, n, of detections obtained over 12 scans for P_{TE} and over 10 scans for $P(TE|TTD)$.

Examination of the coefficients of P_{TE} reveals that the Q-test coefficients more closely match the binomial coefficients than the run-test coefficients; this fact is especially important at the expected values of n. For $P_D = 0.7$, the expected number of detections in 12 scans is 8.4. The coefficient for R = 5 at n = 8 is only 35 percent of the maximum value, while the corresponding Q-test coefficients are 99 percent of the binomial coefficients. At n = 9, the run-test coefficients range from 18 to 64 percent of the binomial coefficients, while the Q-test coefficients are at least 97 percent of the maximum value. Thus, from the standpoint of track establishment for real targets, the Q-tests enjoy a significant advantage because of the more efficient weighting given to the most likely detection sequences for values of P_D in the range 0.7 to 0.9. For values of P_D above 0.9, the differences in weights are less significant. For R = 7, however, the differences are still significant at $P_D = 0.95$.

Examination of the coefficients for $P(TE|TTD)$ reveals that although the leading terms for the run tests and the most nearly equivalent Q-tests are identical, the higher order coefficients for the Q-test coefficients are substantially greater than those for the run tests. Thus, the most nearly equivalent Q-tests will exhibit

*Under the column heading K/12.

TABLE 19. COMPARISON OF POLYNOMIAL COEFFICIENTS

Test Parameters			P_{TE}								$P(TE \mid TTD)$							
R	Q_+/Q_c	K/12	5	6	7	8	9	10	11	12	3	4	5	6	7	8	9	10
5			8	49	126	175	140	60	12	1	1	7	21	35	35	21	7	1
	2/9		8	112	496	491	220	66	12	1	1	19	138	203	119	45	10	1
	4/15		8	274	663	491	220	66	12	1	1	67	231	203	119	45	10	1
	6/21		8	427	663	491	220	66	12	1	1	175	231	203	119	45	10	1
		5	495	924	792	495	220	66	12	1	120	210	252	210	[illegible]	45	10	1
6				7	36	75	80	45	12	1		1	6	15	20	15	6	1
	2/11			7	112	421	220	66	12	1		1	24	203	119	45	10	1
	4/19			7	339	454	220	[illegible]	12	1		1	121	203	119	45	10	1
	6/27			7	449	454	220	[illegible]	12	1		1	231	203	119	45	10	1
		6		924	792	495	220	66	12	1		210	252	210	120	45	10	1
7					6	25	40	30	10	1			1	5	10	10	5	1
	2/13				6	105	214	66	12	1			1	30	119	45	10	1
	4/23				6	332	214	66	12	1			1	203	119	45	10	1
	6/33				6	332	214	66	12	1			1	203	119	45	10	1
		7			792	495	220	66	12	1			252	210	120	45	10	1

significantly higher false-track establishment rates than the run tests, unless P_2 is quite small. The extreme example of this is obtained from comparing $R = 7$ with $Q_+ = 6$, $Q_o = 33$. This Q-test will exhibit at least twice the false-track-establishment rate of the run test unless P_2 is less than 0.005, which corresponds to P_{FD} being less than 0.00056 for $N_2 = 9$.

This is not to say that the performance superiority of the Q-tests is in doubt. A single example will illustrate this point. The run test $R = 6$ will be compared with the Q test $Q_+ = 4$, $Q_o = 26$ for $S = 12$. The difference in the probability of track establishment is given by

$$P_{TE}\ (Q_+ = 4,\ Q_o = 26) - P_{TE}\ (R = 6) =$$

$$P_D^{10}\ (1 - P_D)^2 [21 + 108\ x - 46\ x^2 - 36\ x^3 - 7\ x^4] \tag{25}$$

where

$$x = (1 - P_D)/P_D .$$

This difference is positive (and P_{TE} for the Q-test is greater than P_{TE} for the run test) if the bracketed quantity in Eq. (25) is positive, which is clearly the case if x is sufficiently small. The smallest positive root of the bracketed quantity is $x = 1.235$, which means that P_{TE} for the Q-test is superior to P_{TE} for the run test if $x < 1.235$, or if $P_D > 0.45$. At $P_D = 0.45$, the value of P_{TE} for the run test is about 0.036, so that the Q-test is superior to the run test in track-establishment performance against real targets for any useful value of P_D.

Next, the difference in the conditional probability of false-track establishment is given by

$$P(TE|TTD,R) - P(TE|TTD,\ Q) =$$

$$P_2^9\ (1 - P_2)\ [y^5 + 6\ y^4 + 3\ y^3 - 99\ y^2 - 30\ y - 4] \tag{26}$$

where

$$y = (1 - P_2)/P_2$$

This difference will be positive (and the average false-track establishment rate for the run test will be greater than that for the Q-test) if y is sufficiently large. The largest positive root of the bracketed quantity in Eq. (26) is $y = 3.059$, which means that the run-test false-track establishment performance is inferior to the Q-test if $P_2 < 0.246$. For $N_2 = 9$, $P_2 = 0.246$ implies $P_{FD} = 0.039$, and the average false-track establishment rate at the cross-over point is found to be $R_{FTE} = 14$ false established tracks per minute. Thus, the run test yields inferior false-track establishment performance to that of the Q-test whenever the false detection rate is sufficiently low to yield practically useful values of R_{FTE}. However, it is to be noted that the processor burden for generating subsequent correlation windows with the Q-test is 3 to 8.4 times that of the run test under these conditions.

This example illustrates the point that it is possible to find a Q-test that is, for all practical purposes, uniformly superior to a given run test, from the standpoint of track-establishment performance. The price paid is one of increased burden for the track-establishment processor, but the added burden is primarily quantitative in nature, because the association and decision algorithms needed for the Q-tests are only slightly more complicated than those required for the run tests.

D. RECAPITULATION

The nominal implications of the results obtained above and in the previous section will be reviewed in the following operational context. It is again assumed that the surveillance sensor scans 480,000 resolution cells, taking 90 seconds per scan. The correlation windows are assumed to contain $N_1 = 32$ and $N_2 = 9$ resolution cells for initial and subsequent correlations, respectively. It is assumed that 1 percent of the resolution cells will contain sources of fixed-target detections and that the random false-detection probability is $P_{FD} = 0.01$. It is assumed that the probability of a detection report from a single source of fixed-target detections is 0.75.

The processor will, therefore, receive an average of 8400 detection reports per scan, or about 93 per second. If the surveillance sensor is situated on a moving platform, coordinate transformations will be required for each report, for the purposes of both fixed-target-removal processing and association with tentative and established tracks.

The results presented earlier* indicate that the probability of desensitization (loss of a real-target detection report because of prior occurrences of random false detections in the same resolution cell) is of the order of 0.03 for fixed-target-removal processing with $k = 2$, $n = 4$, and less than 0.001 for $k = 3$, $n = 5$. For $k = 4$, $n = 6$, the probability of desensitization is completely negligible. The data-storage burden for fixed-target-removal processing will be greater for $k = 3$, $n = 5$ or $k = 4$, $n = 6$ than for $k = 2$, $n = 4$, but the diminution of the effective probability of real-target detection with $k = 2$, $n = 4$ can reduce the probability of track establishment by as much as 10 percent. For the sake of discussion, it will be assumed that the fixed-target-removal processing parameters are chosen to be $k = 3$, $n = 5$.

The contribution of random false detections to the fixed-target-removal processing burden must be taken into account. This contribution is the same for all three of the aforementioned sets of processing parameters. The expected number of tentative and established fixed-target locations which must be stored in the removal processor because of random false detections is 14,260. The average number of locations which must be stored for actual sources of fixed-target detections is 4725, so that the total fixed-target-removal processing burden is very nearly 19,000 fixed-target-removal associations per scan, or about 211 per second.

*Table 1, p. 32.

The random false detections and residual fixed-target detections result in an effective value of P_{FD} (as seen by the track-establishment processor) of 0.01038. The average number of such false-detection reports received by the track-establishment processor will be very nearly 5000 per scan, or 55 per second. The effective value of P_{FD} results in $P_1 = 0.2839$ and $P_2 = 0.08965$. The average burden for generating initial correlation windows is 3880 windows per scan, or about 43 per second.

The processing burden at this point is summarized in Table 20. This part of the total burden is independent of whether run tests or Q-tests are employed for track-establishment processing. None of the operations listed in Table 20 require prediction of the target location on the next scan. The coordinate-conversion operation can be simplified or perhaps eliminated, and the fixed-target-removal burden can be substantially reduced, if the surveillance sensor is not on a moving platform. By using an essentially invariant catalog of fixed-target locations, this part of the burden can be reduced to an average of 4800 data-storage locations and 53 operations per second. It is of interest to note that if fixed-target-removal processing were not employed, the burden for initial correlation window generation would be increased by 50 percent, and the effective value of P_{FD} seen by the track-establishment processor would be 0.0175.

TABLE 20. PROCESSING BURDEN PRIOR TO TENTATIVE-TRACK DECLARATION

Operation	Data Storage Burden	Average Number of Operations	
		per scan	per sec
Input Buffering, Coordinate Conversion	---	8,400	93
Fixed-Target Removal Association and Updates	20,000	19,000	211
Initial Correlation-Window Generation	4,000	3,880	43
Total	24,000	31,280	347

Table 21 presents values of the average false-track establishment rate and the probability of track establishment obtained using the run tests. The average processor burden imposed by use of the run tests is 1210 subsequent correlation windows per scan, or about 13 per second. Table 22 presents corresponding results for the Q-tests, for $Q_i = 3$, $Q_+ = 4$. The average processor burden imposed by use of the Q tests is 5450 subsequent correlation windows per scan, or 61 per second, for 12-scan track establishment; and 5750 subsequent correlation windows per scan, or 64 per second, for 18-scan track establishment. Thus, the Q-tests require about five times as much processing as the run tests, but the burden in either case is a relatively small part of the total burden prior to tentative-track processing exhibited in Table 20. Comparison of the results presented in Tables 21 and 22 indicates that the added burden for Q-test processing of tentative tracks yields a clear-cut advantage over run-test processing if the single-scan probability of detection is less than 0.9.

TABLE 21. RUN TEST RECAPITULATION

		P_{TE} for $P_D = 0.7$			P_{TE} for $P_D = 0.8$			P_{TE} for $P_D = 0.9$		
R	R_{FTE}	S=6	S=12	S=18	S=6	S=12	S=18	S=6	S=12	S=18
4	5.3 per min	.384	.716	.869	.573	.890	.981	.787	.983	.998
5	.53 per min 32 per hour	.219	.502	.686	.393	.739	.892	.650	.931	.988
6	2.85 per hour	.118	.329	.498	.262	.577	.767	.531	.850	.957
7	.26 per hour 6.1 per day	---	[illegible]	[illegible]	---	[illegible]	.675	---	.717	.899
8	.55 per day	---	.127	.278	---	.302	.351	---	.603	.822

TABLE 22. Q-TEST RECAPITUALTION

	S = 6				S = 12				S = 18			
		P_{TE} for P_D equal to				P_{TE} for P_D equal to				P_{TE} for P_D equal to		
Q_c	R_{FTE}	0.7	0.8	0.9	R_{FTE}	0.7	0.8	0.9	R_{FTE}	0.7	0.75	0.8
15	0.53 per min	0.218	0.393	0.650	2.8 per min	0.878	0.976	0.999	4.2 per min	0.946	0.996	0.999
18	2.8 per hr	0.118	0.262	0.531	0.81 per min	0.814	0.955	0.997	2.2 per min	0.936	0.993	0.999
20	---	---	---	---	0.20 per min	0.736	0.925	0.994	0.86 per min	0.926	0.989	0.998
25					0.63 per hr	0.524	0.808	0.974	5.0 per hr	0.883	0.971	0.992
29					1.5 per day	0.339	0.641	0.917	0.67 per hr	0.825	0.940	0.981
30					0.47 per day	0.278	0.581	0.895	0.38 per hr	0.804	0.927	0.976
31					0.21 per day	0.237	0.528	0.863	6.0 per day	0.789	0.916	0.971

V. PERFORMANCE IMPLICATIONS

A. A RESOURCE ALLOCATION PROBLEM

The performance of the surveillance system has been described in terms of the mean false-track establishment rate, R_{FTE}, and the probability of track establishment for real targets, P_T. There is a trade-off between these two quantities which can be exercised at the surveillance sensor (modifying the detection criteria so as to change the false-alarm rate and single-scan probability of detection), at the processor (modifying the threshold Q_0), or both. The trade-off can be employed to maximize the value of the output of the surveillance system to the user.

In general, when a track is established, the user will react in some way. If a tangible resource is committed or expended, the reaction will entail a cost to the user, which will be denoted by the symbol C_R. Next, a real target may be missed, meaning that the target appears within the coverage domain of the surveillance system but no resource is expended. This can occur either because the surveillance system fails to establish a track on the target, or because the available resources have been exhausted. The cost to the user for a missed target will be denoted by C_M. Finally, if a real target appears, its track is established, and a resource is expended, there is presumably a gain to the user which will be denoted by G. The reaction cost C_R can be made the same for both false and real tracks, any difference being absorbed in G. Thus, if a false track occurs, the cost to the user is C_R. If a real target is missed, the cost is C_M. If a real target is not missed, the net payoff to the user is $G - C_R$.

In the absence of constraints on the expenditure of resources, the net expected value to the user (assuming that costs and gains

Preceding page blank

accumulate linearly) is

$$V_\infty = N_R P_{TE} (G - C_R) - N_R C_M (1 - P_{TE}) - N_F C_R$$

where N_R is the expected number of real targets occurring during the time period for which the value is computed, and N_F is the expected number of false tracks during that time. If no resources were expended, the expected cost to the user would be

$$C_O = N_R C_M$$

so that the net payoff for expending resources is

$$V_\infty + C_O = N_R P_{TE} (G + C_M - C_R) - N_F C_R$$

B. CONSTRAINED RESOURCES

The effect of resource constraints on the value to the user depends on the nature of the constraints. One such constraint will be considered here. It is supposed that the resources are periodically replenished to a level M. The time period between replenishments will be referred to as an epoch, and the duration of an epoch will be denoted by T_E, and the expected value to the user per epoch will be denoted by V_M. The net payoff to the user per epoch is then $V_M + C_O$. During an epoch, the user can expend up to M resources.

It is assumed that the number of occurrences of real targets during an epoch is a random variate governed by the Poisson distribution with parameter N_R. It is assumed that the targets are statistically independent, insofar as track establishment is concerned. Thus, the conditional probability distribution of the number of real-target tracks that are established (given the number of real targets) is governed by the binomial distribution with parameter P_{TE}. Next, it is assumed that the number of false tracks per epoch is a random variate governed by the Poisson distribution with parameter $N_F = R_{FTE} T_E$.

Finally it is assumed that the occurrences of false established tracks and tracks established for real targets are intermingled homogeneously in a statistical sense; this simply means that for given numbers of false tracks and real-target tracks during an epoch, all time sequences during the epoch are equally likely.

With regard to the last assumption, it is to be noted that if the occurrences of real targets are clustered toward the beginning of the epoch, the results obtained will be conservative, because the user will have less opportunity to waste resources on false tracks before they have been allocated to real targets. Conversely, the results will be optimistic if the occurrences of real targets are clustered toward the end of the epoch, because the resources for that epoch will have been depleted by allocations to false tracks.

It is shown in Appendix B that the net payoff to the user per epoch is given by

$$V_M + C_O = f_M \left(N_F + N_R P_{TE}\right) \left[N_R P_{TE} \left(G + C_M - C_R\right) - N_F C_R\right]$$

where

$$f_M (X) = e^{-X} \left[\sum_{m=0}^{M-1} X^m/m! + M \sum_{m=M}^{\infty} X^m/(m+1)! \right]$$

For $M > 2.5X$, $f_M (X)$ is approximately unity; thus, the resource-constrained case yields a net payoff which is approximately the same as that obtained for the unconstrained case if $M > 2.5 \left(N_F + N_R P_{TE}\right)$.

The criterion for a positive net payoff per epoch is that

$$N_R P_{TE} \left(G + C_M - C_R\right) > N_F C_R$$

which is identical to the criterion without the resource constraint. The resource-constraint factor, $f_M \left(N_F + N_R P_{TE}\right)$ reduces the net payoff when the above criterion is met, but reduces the losses when the net payoff is negative because of high false-track establishment rates.

A numerical example will be given to illustrate this last point. The values $G = 3\ C_R$, $C_M = 3\ C_R$ will be assumed. A positive payoff per epoch will then be realized if

$$5\ N_R\ P_{TE} > N_F$$

It is now supposed that the duration of an epoch is two hours, and that targets will be in the surveillance domain for twelve scans. Table 23 lists the pertinent parameters for the net payoff computation as a function of the track-establishment threshold, assuming $P_D = 0.75$, $P_{FA} = 0.01$, and $Q_+ = 4$. The values of the net payoff per epoch are normalized with respect to C_R, and are given for the unconstrained case and for $M = 4$. It is assumed that $N_R = 2$.

TABLE 23. NET PAYOFF PER EPOCH

Q_o	P_{TE}	R_{FTE} (per hour)	$N_R\ P_{TE}$	N_F	$\frac{V_\infty + C_o}{C_R}$	$\frac{V_4 + C_o}{C_R}$
21	.819	5.09	1.64	10.18	-1.98	- .67
22	.793	3.24	1.59	6.48	+1.47	+ .72
23	.772	2.63	1.54	5.26	+2.44	+1.39
24	.715	1.10	1.43	2.20	+4.95	+4.15
25	.672	.46	1.34	.92	+5.78	+5.48
26	.623	.22	1.25	.44	+5.81	+5.68
27	.592	.15	1.18	.30	+5.60	+5.51

The tabulated payoffs indicate that for the assumed conditions, the payoff will be negative for $Q_o \leq 21$. The resource constraint reduces the loss per epoch by a factor of almost 3 at $Q_o = 21$, but at $Q_o = 24$, the resource constraint only reduces the payoff per epoch by 16 percent. Finally, the optimum value of Q_o is seen to be 26, but the constrained payoff obtained is within 94 percent of the unconstrained maximum if Q_o differs from the optimum value by ± 1.

Finally, it is to be noted that the net payoff for a "perfect" surveillance system ($P_{TE} = 1$, $N_F = 0$) for this example would be simply N_R $(G + C_M - C_R)$, which would be equal to 10 C_R for the values assumed. The realizable system treated in the example, therefore, attains a net payoff which is 57 percent of that of the perfect system with M = 4, and 58 percent without resource constraints.

VI. RELATED PROBLEMS AND ISSUES

A. MULTI-SENSOR CORRELATION

1. Some Generalities

The preceding discussions have dealt primarily with the track-establishment process for surveillance data obtained from a single scanning sensor. The sensor (e.g., a helicopter-borne surveillance radar) is assumed to report the instantaneous locations of apparent targets moving on the surface of the earth without scan-to-scan integration. For several reasons, the use of multiple inputs to the track-establishment process warrants consideration.

First, when more than one radar is employed, the simultaneous coverage available to the system is obviously enhanced. For helicopter-borne radars, the possibility exists of continuing tracks initiated or established by the data from one radar with the data from another at a later time. If the radars are operated with overlapping coverage, the available data rate may be effectively increased in the joint coverage regions. Alternatively, the coverage provided may be complementary, when targets which are masked by terrain from one radar are exposed to the other and vice versa.

The possibility also exists for employing nonscanning sensors as adjuncts to one or more radars. Of particular interest are presence-determination sensors such as remote intrusion-detection devices, which could be employed to fill in coverage gaps of the radars due to foliage masking, or to provide target-classification or threat-ordering information not available from the radar.

Preceding page blank

It is to be noted, however, that the introduction of multiple sensors in the track-establishment process introduces complications, not the least of which is that of achieving satisfactory registration. In this context, registration refers to the degree to which the locations associated with detections of the same target by different sensors will agree sufficiently well to permit the detections to be mutually associated. The problem of registration applies to discrete clutter elements (fixed targets which are detected as moving targets by the radar) as well as to moving targets. The manner in which the registration problem influences fixed-target removal is not the same as the influence on moving-target-track establishment, however.

Other complications result from the possibility of overlapping coverage by two or more scanning sensors; the desirability of maintaining tracks initiated or established by one patrolling sensor when a subsequent patrolling sensor is covering the region through which the track is moving; and the task of introducing presence-determination data from nonscanning sensors. These points will now be discussed.

2. Similar Sensors

The first situation to be considered is that in which two or more scanning sensors are providing data to the track-establishment processor. The simplest (and least interesting) case here is that in which the coverage regions of the individual sensors never intersect, either simultaneously or at different times. In this instance, the complication is simply additive; the processor deals with each of the sensor inputs independently.

The next case is that in which two scanning sensors have overlapping coverage. Thus, suppose that two radars each provide coverage over a circle of radius R and are separated by a distance D, with $0 \leq D \leq 2R$. The fraction of the total area covered by the two radars which is jointly covered is given by

$$A_0/A_T = f(x)/[1 - f(x)]$$

where

$$f(x) = (1/\pi)\left[\arccos x - x\sqrt{1 - x^2}\right]$$

and

$$x = D/2R$$

Table 24 gives values of A_0/A_T as a function of D/2R

TABLE 24. FRACTIONAL OVERLAPPING COVERAGE

D/2R	A_0/A_T
0	1
0.1	0.774
0.2	0.596
0.3	0.453
0.4	0.337
0.5	0.243
0.6	0.166
0.7	0.104
0.8	0.055
0.9	0.019
1.0	0

Thus, if the radars each have a range of 20 km and are separated by a range of 16 km (D/2R = 0.4), then about one-third of the total surveillance area is covered* by both radars.

The question then arises as to what should be done with apparent target-detection reports from the two radars in the overlap region. The simplest solution is simply to ignore the detection reports from one of the two radars in the overlap region; this can be accomplished

*This statement ignores the effects of masking due to terrain or foliage. However, if the masked regions are small compared to the overlap area, and uniformly distributed, then the ratios of Table 24 are still valid.

by deleting reports from the data transmitted by each radar to the track-establishment processor either at the radar itself or in the processor. However, it is to be noted that in the overlap region, the effective data rate available to the processor is just the sum of the data rates from the individual sensors. Thus, for two radars operating at simultaneous scanning rates, it should be possible to reduce the time required to establish tracks by one-half. At this point, it is necessary to consider the class of track-establishment procedures being used by the processor.

A major complication from display-integration procedures (such as the time-compression technique) arises from the fact that the inputs from the multiple sensors will generally not be synchronized. It is therefore unclear as to how nonuniformly spaced inputs can be stored by the display processor for effective integration by the operator. The scan-to-scan correlation techniques that have been analyzed in this report do not depend on synchronism, however. The tentative-track declaration process and the automatic track-establishment can be employed with asynchronous inputs.

Considering the fact that moving-target detectability for a radar depends on the range-rate exhibited by the target to the radar, the use of two radars with overlapping coverage should enhance the detectability of targets moving along meandering routes. That is, a target moving with a speed well above the minimum detectable range rate cannot simultaneously exhibit a low range rate to both radars. A target moving along a meandering route will be seen by one radar and then the other; the result of combining the two sets of detection reports may result in track establishment where such would be unlikely on the basis of reports from a single radar.

With regard to fixed-target-removal processing, the tentative conclusion reached in this study is that such processing should be done on an individual-radar basis*. The justification of this

---

*This does not preclude the possibility of maintaining a library of well-established fixed targets (which consistently yield detection reports) in the processor.

statement is that many discrete clutter targets are strongly aspect-dependent, both as to the effect of the target echo on the radar processor and as to the apparent location of the resulting detection report from a given radar. Thus, two radars may not both report detections from a fixed target, and if they do, they may report different apparent locations of the target because of differences in the geometry of the back-scattering process.

The third case to be discussed here has to do with patrolling sensors, in which a region which was covered by one sensor is now being covered by the other. The scan-to-scan correlation processing techniques considered in this paper enable the continuance of a track which has been established on the basis of reports from the earlier sensor on the basis of detection reports received subsequently, without requiring reestablishment by the later sensor.

Assuming that the coverage of the later sensor occurs soon enough after the coverage of the earlier sensor, the tentative tracks declared on the basis of reports by the earlier sensor can be continued and established using the reports of the later sensor. This capability effectively increases the available scan-to-scan integration time for a given patrol speed, and therefore may significantly increase the probability of track establishment and/or permit greater patrol speeds.

This admittedly qualitative discussion has indicated in a general way some of the advantages to having multiple-sensor inputs to the track-establishment processor. To recapitulate, the advantages cited include (in addition to the obvious one of greater geographical coverage) the possibilities of faster track establishment, more reliable track establishment against targets following meandering routes, longer track life and higher patrol speeds. In addition, it may be possible to achieve more precise target locations under certain conditions. The potential for realization of these benefits, on the other hand, is strongly dependent on the kind of processing employed for track establishment. Apart from processor sizing considerations, the automatic scan-to-scan correlation procedures are insensitive to the use of multiple inputs. Manual (display-integration) techniques are not as well understood in this regard, because of the question of dealing with asynchronous inputs.

3. Presence-Determination Inputs

This section presents a discussion of possible interactions between a fixed or patrolling scanning sensor, the track-establishment processor and an array of fixed presence-determination sensors. Examples of presence-determination sensors include remote seismic and acoustic intrusion detectors and various types of "trip-wire" devices, including remote photoelectric sensors which report when a light beam is interrupted. In their most rudimentary form, presence-determination sensors provide an indication that a target of interest is proximal to the sensor itself, of has traversed a region proximal to the sensor. More complex configurations provide target signatures which may be useful for target enumeration or classification. Such devices are usually adapted to the intended operational environment. A properly emplaced seismic sensor, for example, will provide target-presence determinations regardless of the weather conditions, and is not affected by foliage masking.

Conversely, the coverage provided by the types of presence-determination sensors considered here is usually quite limited (e.g., to ranges of a few hundred meters) by comparison with possible helicopter-borne search radars, which may have ranges of 20 kilometers. Typically, individual presence-determination sensors do not provide data as to the direction of movement of the target; however, such data can be inferred by associating multiple reports from an array of such sensors. It is possible to deal with the track-establishment problem solely in terms of arrays of presence-determination sensors, but a detailed discussion is beyond the scope of this paper. It will suffice to say that coverage of a large perimeter in sufficient depth to permit target tracking for a sufficiently long period of time may require many hundreds of sensors, with attendant problems of providing communication links between the individual sensors and the processor, and of providing effective means for integrating the sensor outputs. Nevertheless, such an approach may be attractive in heavily foliated areas where target detection or tracking by helicopter-borne search radars is not possible.

The viewpoint taken in this discussion is to regard the scanning sensor and the fixed-sensor array as complementary and mutually supporting elements of the surveillance process. The scanning sensor provides large-area coverage over open regions and periodically updated target-position information. The fixed-sensor array provides complementary coverage in masked regions and data for threat ordering. In this context, tracks established via reports from the scanning sensor can be employed to control the monitoring process for the fixed-sensor array. Conversely, detection reports from elements of the fixed-sensor array can be employed to initiate or assist in establishing tracks based on reports from the scanning sensor, and can provide continuity for such tracks in masked regions.

The manner in which inputs from the presence-determination sensors are used with scan-sensor reports in the tracking process deserves comment. The integration process can be accomplished in several ways, and the choice depends on the confidence which is attributed to the presence-determination reports. Assuming that the frequency of real targets is low (e.g., less than 10 per hour) and that the false-alarm rate from the fixed-sensor array is low (e.g., less than 10 per hour), integration can be accomplished by the processor-display operator. On receiving a detection report from the fixed-sensor array (displayed at the location of the reporting sensor with appropriate means for indicating the uncertainty in target location if this is greater than the corresponding uncertainty for scanning-sensor reports), the operator would first call for a display of any tentative or established tracks with the most recent report in the neighborhood of the presence-determination input. Based on the apparent quality of the tentative track and the confidence in the presence-determination input, the operator could declare the track to be established, or increase its quality number by a specified amount ΔQ_{+}^{*}.

In the absence of a tentative or established track which can be associated with the presence-determination report, the operator can use the presence-determination report to initiate a track, with the initial quality number Q_{i}^{*} determined by the confidence in the report and the size of the initial correlation window determined by the uncertainty in the apparent target location. Subsequent detection reports from the scanning sensor would be employed to develop a tentative or established track history in the usual manner.

The same procedures could be accomplished automatically. Such a configuration would be essential if the total false-alarm rate from the fixed-sensor array exceeded the capabilities of the operator. Such inputs would not be subjected to the fixed-target removal procedure†, but would be associated with existing tentative or established tracks or used to initiate new tracks.

An alternative approach exploits a feature of certain presence-determination sensors, which is the capability to store detection reports; readout from such sensors is accomplished via a command link. In this case, the readout process could be controlled via the occurrence of apparent tracks (based on reports from the scanning sensor) passing in close proximity to elements of the fixed sensor array. This approach would tend to minimize the channel capacity required for readout from the array, since an element is interrogated only if (and when) a tentative or established track is passing through the coverage region for that element.

4. Other Interfaces

It is also possible to regard the scanning-sensor/track-establishment configuration as a means for vectoring specialized

† Certain procedures for the removal of spatially correlated false alarms might be needed, however.

mobile sensors, whose function would be to provide track condirmations, threat descriptions, and improved target localization. Sensors of this type would include imaging electrooptical devices, which can provide a visual picture of the target. The field of view and other limitations of these sensors tend to preclude their use for area surveillance. Used in conjunction with real-time target-tracking data, however, they could be directed to examine relatively small regions.

B. MAP CORRELATION

1. Concepts

There are several ways in which map data can be employed in conjunction with scanning sensor output. The simplest of these permits the processor to ignore inputs from regions in which targets either will not be detected, or in which any targets detected are of no interest. Thus, inputs would be ignored when the sensor is scanning regions which are known to be target-free (impassable terrain), in which targets will be masked (by foliage, for example), on regions known to contain only targets which are of no interest to the surveillance-data user (e.g., areas containing only nonhostile targets). The effect of such deletions in the processor input is to reduce the computational load for track establishment, and, if the deleted regions represent a significant fraction of the total area under surveillance, a reduction in the overall false-track establishment rate.

The censoring process just described could be controlled manually by providing the operator with means (e.g., a light pen) to define the regions to be ignored by the processor. (The same feature would permit the operator to delete regions in which targets cannot be detected because of background or clutter conditions.) Alternatively, preselected regions to be ignored could be defined in terms of polygons for storage in the processor. Such features could also be employed to define regions in which targets cannot be detected because of background conditions.

Another map correlation technique is to modify the processor track-establishment criteria according to the geographical relationship of areas under surveillance to key points. Thus, less stringent criteria would be used for areas where early track establishment is important; for example, because hostile forces in such areas would jeopardize installations or friendly force elements.

From the standpoint of track establishment as such, perhaps the most important map correlation technique is that which exploits knowledge of routes over which targets are likely to move in the scan-to-scan correlation process. If the surveillance region can be confined to a known route structure, then several benefits accrue. Such a route structure can be stored in the processor as connected sequences of line segments*. The route structure is obtained from maps or from reconnaissance imaging. In principle, it is possible to infer unknown segments of the route structure from multiple track histories of targets which traverse the unknown segments.

The next section presents some brief analyses which will indicate the potential benefits of route correlation techniques.

2. Analysis

In the preceding analyses of track establishment processing, the scan-to-scan correlation process was considered in absence of any prior knowledge as to most likely locations or directions of movement of targets. Accordingly, the correlation window dimensions were dictated by the requirement to accommodate targets that might be located at any point in the region under surveillance, and which might move in any direction. However, in order to establish a bound on the false scan-to-scan correlation probability, it was necessary to set an upper bound on the rate of movement of the target. For a scan rate of 90 seconds and a sensor resolution cell 100 feet by 500 feet, a target moving at a maximum speed of 5.56 ft/sec traverses a maximum distance of about 500 feet from scan to scan. Without route correlation, the correlation window for tentative-track declaration must

*Breaks in the sequences may occur in masked regions.

contain 32 resolution cells* to accommodate all possible directions of movement. For a single-scan probability of false alarm per resolution cell of 0.01, the probability that a false alarm on one scan is falsely correlated (resulting in tentative track declaration) on the next scan is 0.275. Following tentative-track declaration, the correlation window is reduced to 9 cells, and the false-correlation probability is 0.0865.

For the sake of discussion, it will now be assumed that the target is known to move on a route which traverses the resolution cells as shown in Fig. 3. Given that an apparent target is initially detected in the cell marked I in Fig. 3, three options are available. If the direction of motion of the traffic along the route is not known, 32 cells can be arranged as shown by the shaded area in Fig. 3. Under these conditions, a target will fall in the tentative track declaration window if it is moving in either direction along the route with a speed less than 13.4 ft/sec (9 mph), or more than double that achievable without map correlation. If only those targets moving in a specified direction are of interest, the speed range can be doubled again. These options will yield the same scan-to-scan false-correlation probabilities as were obtained with the 32-cell window originally.

Alternatively, if the target moves no faster than 5.56 ft/sec, the correlation window for tentative-track declaration can be reduced to 18 cells, as shown by the cross-hatched area in Fig. 3. At a single-scan false-alarm probability of 0.01, the tentative-track-declaration, false-correlation probability would be reduced to 0.1655, reducing the processor burden and false-track-establishment rate by at least 40 percent. Again, if there is a preferred direction of target motion along the route, the number of cells in the <u>tentative-track-declaration</u> correlation window can be reduced to 9 (as is employed in the track-establishment window without map correlation), for a false-correlation probability of 0.0865.

*The center cell of the 3x11-cell window is deleted because of the fixed-target-removal feature.

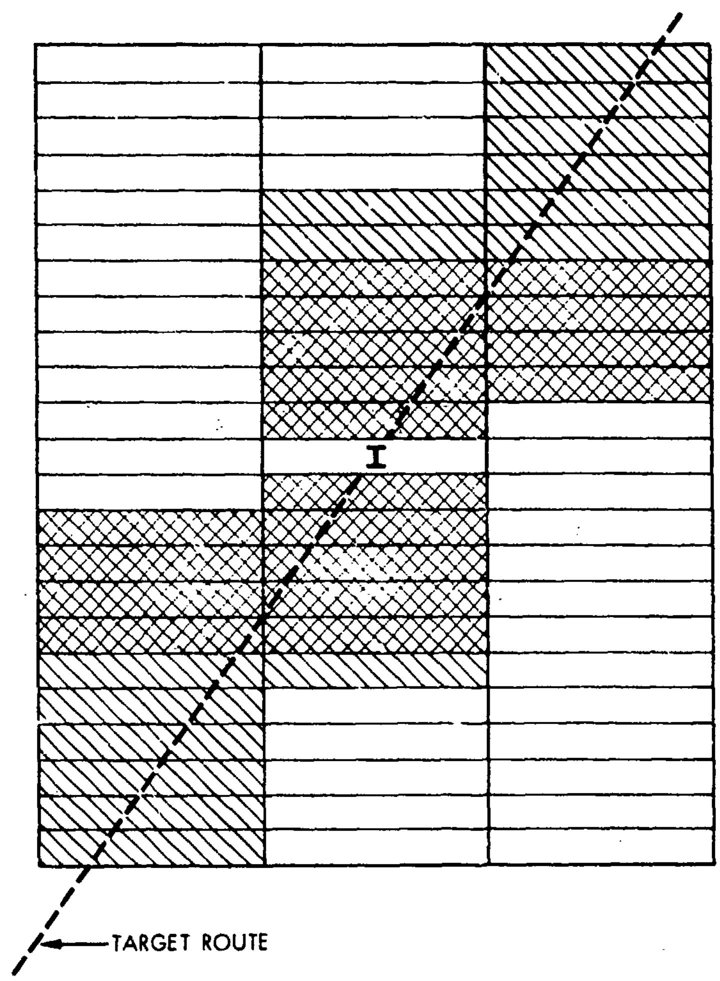

FIGURE 3. Tentative-Track-Declaration Correlation Windows With Map Correlation

Similar benefits result from the use of map correlation on tentative tracks; in some instances, it may be possible to reduce the correlation window to 6 or even 3 cells. The result of such a reduction would be a dramatic reduction in the false-track-establishment rate. For a 6-cell window, the probability of false correlation after a false tentative-track declaration is 0.0585, for a single-cell false-alarm probability of 0.01. Because the false-track-establishment rate varies as the fourth (or higher) power of the false-correlation probability, the reductions indicated would yield at least an eightfold reduction in the false-track-establishment rate. (Alternatively, the false-track-establishment rate can be maintained at a higher single-cell false-alarm probability, e.g., 0.015 in the preceding examples.) These discussions do not take into account the task of processing apparent target detections which do not correspond to known routes. If all or a very large fraction of targets do move along known routes, then this form of map correlation may be of great benefit. Conversely, such map correlation could seriously degrade the performance of the track-establishment process if targets are likely to depart from routes they are expected to follow.

C. MIXED TARGET TYPES

1. Consequences for Tentative-Track Declaration

The preceding discussions have dealt with the problems of establishing tracks on targets which exhibit a minimum detectable rate of motion and are bounded by some constraint with respect to maximum speed. The specification of a minimum rate of motion, it will be recalled, interacts with the task of fixed-target removal. In order to be established as a track, the target must move at least one resolution cell during the time employed for fixed-target removal: one scan period in the simplest instance. This suggests that the scan period be tailored to the minimum detectable target speed. Whether or not this is done, the maximum target speed establishes the minimum size of the initial correlation window, to ensure that the target is not lost (insofar as track establishment is concerned) by escaping from the correlation window before the next scan.

These remarks are true in a qualitative sense regardless of the type of target being tracked. In the examples used, a target moving with a minimum speed of 1.5 feet per second (say) is certain to move from one 11-foot resolution cell to another over a scan period of 90 seconds, and therefore would not be rejected by the fixed-target-removal algorithm except by accidental occurrences of false alarms.

(This last contingency is of negligible likelihood for false-alarm rates of a few percent or less.) Conversely, if the target is constrained to a maximum speed of 5.5 feet per second, then an initial correlation window which extends at least 500 feet in each direction from the cell containing the initial detection will ensure that the target is certain to be in the correlation window on the next scan. Against targets moving with speeds between 10 and 36 mph, the scan period could (in principle) be reduced to 9 seconds, and the same considerations would pertain.

The problem of mixed target types arises when one target class determines the minimum scan rate, but another class determines the minimum size of the initial correlation window. Combining the two previous examples, the slower targets would set the minimum scan period at (say) 90 seconds. An initial correlation window which was sufficiently large to ensure the inclusion of a vehicular target on the scan following the initial detection would require minimum linear dimensions ten times as great as those required for detecting slow targets alone. The consequence of this would be a drastic increase in the probability of a false correlation of a false alarm.

As a concrete example, the 32-cell initial correlation window assumed in the analyses is increased to 1994 cells. (From 3 x 11 (500 feet-by-100 feet) cells to 21 x 95 cells; in each instance, the central cell is deleted by the fixed-target rejection criterion.) The probability of a false initial correlation of a false alarm is given by

$$P_1 = 1 - (1 - P_{FD})^{N_1}$$

where N_1 is the number of resolution cells in the initial correlation window, and P_{FD} is the probability of a false detection in a resolution cell on a single scan. It is easily seen that a value of P_{FD} which is tolerable for $N_1 = 32$ is ridiculous for $N_1 = 1994$. At $P_{FD} = 0.01$, N_1 yields a false initial correlation probability of about 0.27. With $N_1 = 1994$, $P_{FD} = 0.01$ yields not only virtually certain false correlation, but the expected number of false detections in the correlation window is about 20. This means that each false detection would lead, on the average, to 20 false tentative tracks, hardly a stable situation from the processor standpoint.

In the example just given, the speed ratio between the two classes of targets was ten to one. The problem is clearly aggravated still further if this ratio is larger, which will be the case if more rapidly moving target types are included in the system surveillance requirements.

One way of offsetting the consequences of a large initial correlation window would be to reduce the false-detection probability. In principle, this can be accomplished in a moving-target-detection radar by means of a dual-channel or multi-channel signal processing scheme. Detections obtained from the processing channel which is associated with the lower speed target class are used only in conjunction with the initial correlation window needed for that class. In the processing channel associated with the higher speed target classes, the minimum detectable rate of motion is increased to provide improved rejection of fixed-target returns and to reduce the false-alarm rate accruing from a variety of sources. However, it should be noted that to obtain equivalent tentative-track declaration statistics at $N_1 = 1994$ as were obtained with $N_1 = 32$, a 60-fold reduction in the false-detection rate must be achieved in the high-speed processing channel output. While substantial reduction may be realizable, such a large reduction may be out of the question.

A considerable benefit results under certain conditions if the sensor reports include a speed estimate for the target that has been detected, and if the scan rate is sufficiently high. The latter condition amounts to saying that the target motion is Markovian, in the sense that the target position on one scan is highly correlated with the target position that would have been estimated on the basis of the position and velocity observed on the previous scan. Even if the sensor only estimates the magnitude of one component of the apparent target velocity, the number of cells in the initial correlation window can be drastically reduced.

Extending the preceding example, suppose that the sensor is able to estimate the magnitude of the rate at which vehicular targets traverse the 100-foot resolution cells with an error which does not exceed 1.5 mph. Thus, a target detection accompanied by a rate magnitude estimate of 22.5 mph has an actual rate of traverse whose magnitude falls between 21 and 24 mph. Under these conditions, the initial correlation window can be collapsed to include only those cells which correspond to movements within the prescribed limits. The number of cells in the initial correlation window falls to N_1 = 210. This reduction, of course, presumes that the vehicular targets maintain nearly constant rates from scan to scan.

A somewhat more recondite approach involves a substantial increase in the sensor scan rate, the most obvious consequence of which is to reduce the size of the initial correlation window for the high-speed target-track-establishment process. A second benefit may be to improve the degree to which the target motion from scan to scan exhibits the desired Markov property. Increasing the scan rate fivefold in the preceding example would immediately reduce the size of the initial correlation window for the vehicular targets from N_1 = 1994 to N_1 = 44.

The capability for tracking personnel targets can (in principle) be retained by using every fifth scan for this purpose. It then follows that fixed-target-removal processing must also be based on every fifth scan.

This does not mean that only one-fifth of the scans are employed for personnel tracking and fixed-target removal, but that the scans which are associated for this purpose are equivalent (modulo 5). Thus, scans 1, 6, 11, ..., 5n + 1 can be processed as one set, in parallel with scans 2, 7, 12, ..., 5n + 2 as a second set; with scans 3, 8, 13, ..., 5n + 3 as a third set; and so on. This means that a personnel target may (and is likely to) appear as a track in more than one set. While the necessary process of set-to-set association has not been examined in detail, it is clear that a basis exists for both improving the probability of track establishment on real targets and for suppressing false tracks. This is perhaps to be expected because the immediate consequence of increasing the scan rate to deal with vehicular targets was to increase the number of detection opportunities for personnel targets.

To summarize, a requirement to establish tracks concurrently on a variety of target types which collectively exhibit a wide range of speeds introduces essential complications from the standpoint of tentative track declaration. To some extent, it may be possible to transfer some of the burden to the input sensor, either by requiring lower false-alarm rates for target classes requiring large initial correlation windows, or by taking advantage of single-scan target-speed estimates that may be available. In any event, a sequence of interleaved processing stages for slow targets can permit the use of higher scan rates which will obviate the limitations for tracking faster targets.

2. Track Establishment and Tracking Implications

After tentative track declaration, the requirement to process mixed target types no longer presents the essential difficulties discussed above. The locations of subsequent correlation windows are predicted on the basis of two detections in different resolution cells, and the differences in target speeds are automatically taken into account. Some secondary effects should be mentioned, however; these arise when the scanning rate is constrained by the slowly moving target class, so that the separation between detections of rapidly moving targets is large compared to the dimensions of the subsequent correlation windows.

When the dimensions of the correlation windows used in track establishment are comparable to the distance traversed by a typical target from one scan to the next, the track establishment process permits a reasonable degree of variability in the target speed and its direction. Greater tolerance of such a variability requires larger subsequent correlation windows. This tolerance of variability carries the penalty of higher false automatically established track rates, which is offset to some extent by the fact that some false tracks will exhibit a meandering characteristic and can be rejected by an operator on this basis.

Conversely, if the track-establishment process must deal with targets which typically traverse great distances (compared to the dimensions of the correlation window) from scan to scan, then little variability in the target velocity can be tolerated, and most of the false tracks will not exhibit a distinctive meandering characteristic.

In the case of personnel targets, use of a 9-cell subsequent correlation window ensures that the track will not be dropped because the target appears outside the correlation window if its speed changes by less than about 20 percent and its direction changes by less than about 40 degrees from scan to scan. In the case of vehicular targets, which can easily move through 20 resolution cells from scan to scan, only a 5 percent speed variation can be tolerated.

REFERENCES

1. P. E. Nathanson, Radar Design Principles, McGraw-Hill, pp. 100-106, 1969.

2. Nelson Wax, "Signal-to-Noise Improvement and the Statistics of Track Populations," J. Appl. Phys., 26, 1955, pp. 586-595.

3. J. V. Uspensky, Introduction to Mathematical Probability, McGraw-Hill, 1937, p. 78.

4. William Feller, An Introduction to Probability Theory and Its Applications, Vol 1, Wiley, 1950, p. 265.

5. William Feller, op. cit, p. 182-183.

APPENDIX A

COMPUTATIONAL TECHNIQUES FOR OBTAINING ANALYTICAL RESULTS

The derivation of formulas for the probability of track establishment and the false-track establishment rate is quite laborious when the number of scans employed is not small ($S_T \geq 6$). In this study, a technique was developed which employs a computer to derive, in effect, these formulas.

Specifically, consider the task of deriving the probability of establishing track on a target which is in view for S scans, given the parameters of the track-establishment process: Q_1, Q_+, Q_-, and Q_o. The possible sequences of detections and misses for the target can be placed into one-to-one correspondence with the binary numbers from 0 to $2^S - 1$. Thus, for the binary number $A_1A_2 \ldots A_S$, let A_m be equal to 1 if the target was detected on the mth scan, and let $A_m = 0$ if the target was missed on that scan. The set of numbers $\{A_1A_2 \ldots A_S\}$ then includes all possible hit-miss sequences in S scans, ranging from 000 ... 00 (no detections) to 111 ... 11 (no misses). The probability of occurrence of a sequence $A_1A_2 \ldots A_S$ is just $P_D^H Q_D^{S-H}$ where H is the number of ones (hits) and $Q_D = 1 - P_D$.

In general, the probability of track establishment in S scans can be written*

$$P_{TE} = \sum_{H=H_{min}}^{S} a_H P_D^H Q_D^{S-H} \qquad (A-1)$$

*This is not the only form for P_{TE}, of course, but it is the most convenient representation for this method.

Preceding page blank

where H_{min} is the minimum number of hits required to establish track (a simple function of Q_o and Q_+), and the coefficients a_H are to be determined.

In its simplest form, the computational technique involves the listing of the 2^S binary numbers. Each number is then analyzed according to the track-establishment algorithm to determine whether the sequence of hits and misses permitted track establishment. This is accomplished by scanning the binary digits $A_1, A_2, \ldots, A_S$ in turn. Tentative track declaration occurs the first time a run of two ones is encountered; then Q is set equal to Q_i. Subsequent ones cause Q to be increased by Q_+, and subsequent zeros cause Q to be decreased by Q_-. If Q drops to zero or below, it is set equal to zero, and the tentative track declaration criteria must then be fulfilled anew if track establishment is to occur. If Q reaches or exceeds Q_o, then the value of a_H (initially set at zero) is increased by 1, where H is the number of ones in the binary number.

When all of the binary numbers have been disposed of in this manner, the numbers a_H represent the number of ways that the track can be established given H hits, which is just what is needed for the representation of Eq. (A -1).

A similar procedure is employed for obtaining the false-track-establishment rate, with the following differences. First, the false-track-establishment rate is written in the form

$$R_{FTE} = R_{FTTD} \quad P(TE \mid TTD) \qquad \text{(A-2)}$$

where R_{FTTD} is the rate of false tentative track declarations, and P (TE|TTD) is the probability that track establishment occurs (without Q dropping to zero or below), given tentative track declaration. For the purposes of this study,

$$R_{FTTD} = \left(\frac{N \quad P_{FD}}{T} \right) \frac{P_2}{1+P_2} \qquad \text{(A-3)}$$

where N is the total number of resolution cells per scan, T is the scan period, and P_2 is the probability of at least one false detection in the initial correlation window.

The probability P(TE|TTD) can be written in the form

$$P(TE|TTD) = \sum_{H=H^*_{min}}^{S-2} b_H \, P_2^H \, Q_2^{S-H} \tag{A-4}$$

where $Q_2 = 1 - P_2$, H^*_{min} is the minimum number of hits (usually H_{min} - 2) required to establish track following tentative track declaration and b_H are coefficients which will be determined by the computer.

The procedure for determining the b_H involves listing the (S-2)-digit binary numbers $A_1A_2 \ldots A_{S-2}$, and analyzing them to determine whether Q, initialized at Q_1, reaches Q_o without first dropping to zero or below. The values of the coefficients b_H are initially set at zero, and b_H is increased by unity if the binary number (containing H ones) meets the criteria just cited. Doing this for all of the 2^{S-2} binary numbers yields the values of b_H for use in Eq. (A-4). Combining Eqs. (A-3) and (A-4) then gives R_{FTE}, as given by Eq. (A-2).

These techniques can be generalized to permit analysis of situations in which the probability of obtaining a hit changes from scan to scan, by computing the probability of each binary number in the list. For such cases, however, it is usually not possible to obtain the compact representations of Eqs. (A-1) and (A-4).

Table A-1 presents the results of the computations for S = 6, $Q_+ = 4$, $Q_i = 3$ in the format employed in the text. The upper set of coefficients are the values of b_H (with the index H replaced by n) for the polynomial P(TE|TTD). Thus, for $Q_o = 10$,

$$P(TE|TTD) = 3 \, P_2^2 \, (1 - P_2)^2 + 4 \, P_2^3 \, (1 - P_2) + P_2^4 \tag{A-5}$$

if $Q_+ = 4$, $Q_i = 3$.

The lower set of coefficients are the values of a_H (again with the index H replaced by n) for the polynomial P_{TE}. Again, for $Q_o = 10$, $Q_+ = 4$, $Q_i = 3$,

$$P_{TE} = 7\ P_D^4\ (1 - F_D)^2 + 6\ P_2^5\ (1 - P_2) + P_2^6 \qquad \text{(A-6)}$$

TABLE A-1. POLYNOMIAL COEFFICIENTS FOR $Q_+ = 4$, 6 SCANS

Q_o	n = 1	2	3	4	5	6
5	3	6	4	1		
6	2	6	4	1		
7	1	6	4	1		
9	0	6	4	1		
10		3	4	1		
11		1	4	1		
14		0	4	1		
15			1	1		
19			0	1		
5	0	0	9	14	6	1
6			7	14	6	1
7			4	13	6	1
9			0	10	6	1
10				7	6	1
11				3	6	1
14				0	5	1
15					2	1
19					0	1

APPENDIX B

PAYOFF EXPECTATION UNDER RESOURCE CONSTRAINTS

A. DEFINITIONS AND ASSUMPTIONS

Let n denote the actual number of false tracks established during an epoch of duration T_E. It is assumed that n is a random variable governed by the Poisson distribution with parameter $N_F = R_{FTE}\, T_E$. Thus, the probability of exactly n false established tracks during an epoch is

$$P\ (n;\ N_F) = N_F^n \exp\ (-N_F)/n! \qquad \text{(B-1)}$$

Let m denote the actual number of real targets appearing in the surveillance domain during an epoch. It is assumed that m is a random variable governed by the Poisson distribution with parameter N_R. The probability of exactly m real targets during an epoch is

$$P\ (m;\ N_R) = N_R^m \exp\ (-N_R)/m! \qquad \text{(B-2)}$$

Let k denote the actual number of real-target tracks established during an epoch. It is assumed that track establishment on a real target is statistically independent of whether tracks were established on other real targets, and whether false tracks have been established. Under this assumption, the conditional probability distribution of k is binomial; the probability of exactly k real-target established tracks, given m real targets is

$$B\ (k;\ m,\ P_{TE}) = \binom{m}{k} P_{TE}^k\ (1 - P_{TE})^{m-k} \qquad \text{(B-3)}$$

 Preceding page blank

The activity during an epoch will be described by the triplet (n, m, k). In the absence of resource constraints, the net payoff, given (n, m, k) is assumed to be given by

$$\Delta V\ (n, m, k) = k\ (G + C_M - C_R) - n\ C_R \tag{B-4}$$

The meanings of G, C_M, and C_R are discussed in the test. The effect of the resource constraint is to limit the number of resources that can be committed during an epoch. The number of resources committed is $n + k$ for $n + k \le M$, and M if $n + k \ge M$.

B. ANALYSIS

The net payoff, for given (n, m, k) and with $n + k \le M$, is given by Eq. (B-4) and is independent of the order of occurrence. The case $n + k \ge M$ will now be considered.

Suppose that there are j real-target established tracks among the first M established tracks. Then the net payoff is

$$\Delta V = j\ (G + C_M) - M\ C_R \tag{B-5}$$

The statistical homogeneity assumption stated in the text means that, given n and k, all sequences of false and real-target established tracks are equally likely. The probability that there are exactly j real-target established tracks among the first M is then given (see Ref. 4) by the hypergeometric distribution:

$$H\ (j; n, k, M) = \frac{\binom{M}{J}\binom{n + k - M}{k - j}}{\binom{n + k}{k}} \tag{B-6}$$

The quantity of interest is the expectation of j; it is shown in Ref. 4 that

$$\sum_{j=1}^{M} j\ H\ (j; n, k, M) = Mk/(n + k) \tag{B-7}$$

If Eqs. (B-4), (B-5) and (B-7) are combined, there is obtained:

$$\Delta V\ (n, m, k) = \begin{cases} k\ (G + C_M) - (n + k)\ C_R & n + k \leq M \\ \dfrac{Mk}{n+k}\ (G + C_M) - M\ C_R & n + k \geq M \end{cases} \tag{B-8}$$

where ΔV has now been averaged over all sequences of n false established tracks and k real-target tracks.

The next step is to calculate the expectation of ΔV, averaging over n and k. It is not hard to show, from Eqs. (B-2) and (B-3), that the unconditional probability distribution of k is the Poisson distribution with parameter $N_R\ P_{TE}$. That is, the probability of exactly k real-target established tracks during an epoch is given by

$$P\ (k; N_R\ P_{TE}) = (N_R\ P_{TE})^k \exp\ (-N_R\ P_{TE})/k! \tag{B-9}$$

The expected net payoff is then to be calculated:

$$\overline{\Delta V} = V_M + C_o = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \Delta V\ (n, m, k)\ P\ (k; N_R\ P_{TE})\ P\ (n; N_F) \tag{B-10}$$

Using Eq. (B-8) in Eq. (B-10) yields

$$V_M + C_o = (G - C_M)\ g\ (N_R\ P_{TE}, N_F, M) - C_R\ f\ (N_R\ P_{TE}, N_F, M) \tag{B-11}$$

where

$$g\ (N_R\ P_{TE}, N_F, M) = \sum_{n+k \leq M}\sum k\ P\ (k; N_R\ P_{TE})\ P\ (n; N_F) + \sum_{n+k > M}\sum \left(\frac{Mk}{nk}\right) P\ (k; N_R\ P_{TE})\ P\ (n; N_F) \tag{B-12}$$

and

$$f\ (N_R\ P_{TE},\ N_F,\ M) = \sum\sum_{n+k \leq M} (n + k)\ P\ (k;\ N_R\ P_{TE})\ P\ (n;\ N_F)$$

$$+ M \sum\sum_{n+k > M} P\ (k;\ N_R\ P_{TE})\ P\ (n;\ N_F) \tag{B-13}$$

Eq. (B-13) can be rewritten, using Eqs. (B-1) and (B-9):

$$f\ (N_R\ P_{TE},\ N_F\ M) = \sum_{m=M+1}^{M} \sum_{n=o}^{m} m \frac{(N_R\ P_{TE})^{m-n}\ N_F^n}{(m-n)!\ n!} \exp\left[-(N_R\ P_{TE} + N_F)\right]$$

$$+ \sum_{m=M+1}^{\infty} \sum_{n=o}^{m} M \frac{(N_R\ P_{TE})^{n-n}\ N_F^n}{(m-1)!\ n!} \exp\left[-(N_R\ P_{TE} + N_F)\right] \tag{B-14}$$

$$= \exp\left[-(N_R\ P_{TE} + N_F)\right] \left\{ \sum_{m=1}^{M} \frac{m}{m!} \sum_{n=o}^{m} \frac{m!}{n!(m-n)!} (N_R\ P_{TE})^{m-n}\ N_F^n \right.$$

$$\left. + \sum_{m=M+1}^{\infty} \frac{M}{m!} \sum_{n=o}^{m} \frac{m!}{n!\ (m-n)!} (N_R\ P_{TE})^{m-n}\ N_F^n \right\} \tag{B-15}$$

The sums over n can be evaluated by invoking the binomial theorem. Doing this gives

$$f\ (N_R\ P_{TE},\ N_F,\ M) = \exp\left[-(N_R\ P_{TE} + N_F)\right] \left\{ \sum_{m=1}^{M} \frac{(N_R\ P_{TE} + N_F)^m}{(m-1)!} \right.$$

$$\left. + M \sum_{m=M+1}^{\infty} \frac{(N_R\ P_{TE} + N_F)^m}{m!} \right\} \tag{B-16}$$

It is therefore seen that f $(N_R P_{TE}, N_F, M)$ is a function of M and $N_R P_{TE} + N_F$.

Operating on g $(N_R P_{TE}, N_F, M)$ in the same manner yields

$$g\,(N_R P_{TE}, N_F, M) = \exp\left[-(N_R P_{TE} + N_F)\right] \times$$

$$\left\{ \sum_{m=1}^{M} \frac{m}{m!} \sum_{n=o}^{m} \frac{m!}{n!(m-n)!} N_F^n (N_R P_{TE})^{m-n} \right.$$

$$+ \sum_{m=M+1}^{\infty} \frac{M}{m!} \sum_{n=o}^{m} \frac{m!}{n!(m-n)!} N_F^n (N_R P_{TE})^{m-n} \qquad \text{(B-17)}$$

$$- \sum_{m-1}^{M} \frac{1}{m!} \sum_{n=o}^{m} n \frac{m!}{n!(m-n)!} N_F^n (N_R P_{TE})^{m-n}$$

$$\left. - \sum_{m=M+1}^{\infty} \frac{M}{m} \frac{1}{m!} \sum_{n=o}^{M} n \frac{m!}{n!(m-n)!} N_F^n (N_R P_{TE})^{m-n} \right\}$$

The first two sums over n can again be evaluated by means of the binomial theorem. The second pair of sums can be evaluated by means of an immediate consequence of the binomial theorem:

$$ma\,(a + b)^{m-1} = \sum_{n=1}^{m} n \frac{m!}{n!(m-n)!} a^n b^{m-n} \qquad \text{(B-18)}$$

The result is that

$$g\ (N_R\ P_{TE},\ N_F,\ M) = \exp\ [-(N_R\ P_{TE} + N_F)]\ \times$$

$$\left\{ \sum_{m=1}^{M} \frac{(N_R\ P_{TE} + N_F)^m}{(m-1)!} + M \sum_{m=M+1}^{\infty} \frac{(N_R\ P_{TE} + N_F)^m}{m!} \right. \tag{B-19}$$

$$\left. - \sum_{m=1}^{M} \frac{m}{m!} N_F\ (N_R\ P_{TE} + N_F)^{m-1} - \sum_{m=M+1}^{\infty} \frac{M}{m!} N_F\ (N_R\ P_{TE} + N_F)^{m-1} \right\}$$

or

$$g\ (N_R\ P_{TE},\ N_F,\ M) = \exp\ [-(N_R\ P_{TE} + N_F)] \left(1 - \frac{N_F}{N_R\ P_{TE} + N_F}\right) \times$$

$$\left\{ \sum_{m=1}^{M} \frac{(N_R\ P_{TE} + N_F)^m}{(m-1)!} + M \sum_{m=M+1}^{\infty} \frac{(N_R\ P_{TE} + N_F)^m}{m!} \right\} \tag{B-20}$$

or, finally,

$$g\ (N_R\ P_{TE},\ N_F,\ M) = \left(\frac{N_R\ P_{TE}}{N_R\ P_{TE} + N_F}\right) f\ \ (N_R\ P_{TE},\ N_F,\ M) \tag{B-21}$$

The original proof of this result was obtained by Dr. Ronald Finkler.

It is convenient to define

$$f_M\ (X) = e^{-X} \left[\sum_{m=o}^{M-1} X^m/m! + M \sum_{m=M}^{\infty} X^m/(m+1)! \right] \tag{B-22}$$

Then

$$f\ (N_R\ P_{TE},\ N_F,\ M) = (N_R\ P_{TE} + N_F)\ f_M\ (N_R\ P_{TE} + N_F) \qquad \text{(B-23)}$$

and

$$g\ (N_R\ P_{TE},\ N_F,\ M) = N_R\ P_{TE}\ f_M\ (N_R\ P_{TE} + N_F) \qquad \text{(B-24)}$$

When Eqs. (B-23) and (B-24) are inserted into Eq. (B-10), there is obtained

$$V_M + C_o = f_M\ (N_R\ P_{TE} + N_F) \left[N_R\ P_{TE}\ (G + C_M - C_R) - N_F\ C_R \right] \qquad \text{(B-25)}$$

which is the result cited in the text.

APPENDIX C

GLOSSARY OF SYMBOLS

Symbol	Meaning	Representative Values
N	Total number of resolution cells examined by the sensor in one scan	$10^5 - 10^6$
P_{FD}	Probability of obtaining a false detection from a given resolution cell on one scan	0.001 - 0.05
T	Time to complete one scan	10 - 120 sec.
P_D	Probability of detecting a given real target on one scan	0.7 - 0.9
N_1	Number of resolution cells in the initial correlation window	32
N_2	Number of resolution cells in the correlation window for tentative tracks	9
Q	Quality number assigned to an apparent target or tentative track during track-establishment processing	---
Q_1	Initial value of Q assigned to an apparent target detection which falls outside of all existing correlation windows	1
Q_+	The amount by which Q is increased when a detection occurs within the correlation window on the next scan	1 - 6
Q_-	The amount by which Q is decreased when no detection occurs within the correlation window on the next scan	1 - 3

Q_i	Value of Q required for tentative-track declaration	$3Q_-$
Q_o	Value of Q required for established-track declaration	$3Q_- + 6Q_+$
S	Total number of scans used for track establishment	6 - 18
P_1	Probability of at least one false detection in the initial correlation window	0.03 - 0.5
P_2	Probability of at least one false detection in the correlation window for tentative tracks	0.01 - 0.2
F	Fraction of false detections obtained during one scan which fall outside of all existing correlation windows	0.7 - 1.0
N_{FT}	Average number of false detections per scan which give rise to false-tentative-track declarations	30 - 2000
R_{FD}	Average false-detection rate	5 - 2500/sec
R_{FTTD}	Average rate of false-tentative-track declarations	1 - 500/sec
R_{FTE}	Average false-track-establishment rate	1/min - 1/day
P_{TE}	Probability of track establishment for a real target	.85
$P(TE \mid TTD)$	Probability of false-track-establishment, given false-tentative-track declaration	---
N_F	Expected number of false established tracks per epoch	---
N_R	Expected number of real targets per epoch	
T_E	Duration of an epoch	2 - 24 hr

G	Incremental payoff for committing a resource against a real target	---
C_M	Incremental cost for not committing a resource against a real target	---
C_R	Cost of a resource	---
M	Maximum number of resources that can be committed during an epoch	---
V_M	Expected value of resource commitment, per epoch, given a constraint of M resources per epoch	---
V_∞	Expected value of resource commitment, per epoch, with no constraint on resources	---
C_O	Expected cost, given no resource commitment	
P_{FTD}	Probability that a source of fixed target detections is detected on a single 0.5 - 1.0 scan	
$\binom{n}{m}$	Binomial coefficient: $\frac{n!}{m!(n-m)!}$	---